

Homework #3

due Wednesday, November 26, 2025

1. (20 pts) Consider the dispersion relation we derived for graphene:

$$E = t \sqrt{1 + 4\cos^2(\sqrt{3}ak_x/2) + 4\cos(\sqrt{3}ak_x/2)\cos(3ak_y/2)}$$

- (a) Expand it in a Taylor series around one of the K-points (for simplicity, the one with $k_x = 4\pi/3\sqrt{3}a, k_y = 0$) to the first order (you may use software of your choice for this).
 - (b) Argue that at around K-points the derivative of E with respect to k should be directly proportional to the Fermi velocity. Using the result of part (a), express the Fermi velocity in terms of the tight-binding transfer integral t and the C-C bond length a (and whatever other constants).
 - (c) Calculate the Fermi velocity given $a = 1.42 \text{ \AA}$ and $t = 2.8 \text{ eV}$.
2. (15 pts) Work through the Nature 2005 and Science 2007 papers by Novoselov et al. (2D gas in graphene and QHE at room temperature, respectively)
- (a) Discuss all differences in behavior of 2D electron gas in graphene as compared to that in 2D behavior in inorganic semiconductor nanostructures
 - (b) What makes it possible to observe QHE in graphene at room temperature?
 - (c) How would Fig.4 in the Nature 2005 paper look like in the case of 2D gas in an inorganic semiconductor?
3. (15 pts) Work through the Science 2009 paper by Gabor et al. Discuss all differences you can find in behavior of carbon nanotube-based p-n junctions as compared to inorganic semiconductor p-n junctions.
4. (10 pts) Work through the Science 2011 paper by Chan et al.
- (a) Discuss differences in MEG in organic semiconductors vs inorganic semiconductor quantum dots (see e.g. M. Beard, in J Phys Chem Lett 2011 for an overview of the latter)
 - (b) Discuss what exactly Figs.2 and 4 tell us.
5. Reading assignment: Nature **462**, 196 (2009) (fractional QHE in graphene).

Additional reading regarding physics of organic materials:

- 1) S. Reich, C. Thomsen, J. Maultzsch, "Carbon nanotubes: basic concepts and physical properties" (Wiley-VCH, Berlin, 2003)
- 2) R. Saito, G. Dresselhaus, M. S. Dresselhaus, "Physical properties of carbon nanotubes" (Imperial College Press, London, 2004)
- 3) M. Pope and C. Swenberg, "Electronic processes in organic crystals and polymers" (Oxford University Press, 1999)
- 4) "Graphene and graphite materials" , H. E. Chan (Ed.), 2010.
- 5) "Physics and chemistry of graphene: graphene to nanographene" , T. Enoki (Ed.), 2010.
- 6) "Perspectives of fullerene nanotechnology", E. Osawa (Ed.), (Kluwer Academic Publishers, 2002)