1. (20 pts) Consider a system containing two spin-1/2 particles. At \( t=0 \), the system is in the state \( |m_{s1}, m_{s2}\rangle = |+, -\rangle \), i.e. \( m_{s1} = +1/2 \), \( m_{s2} = -1/2 \).

The unperturbed Hamiltonian \( H_0 \) is spin-independent and can be taken as zero. At \( t=0 \), a time-independent perturbation is applied \( V = \frac{4\Delta}{\hbar^2} S_1 \cdot S_2 \), where \( S_1 \), \( S_2 \) are spin operators (of the particles 1 and 2, respectively), and \( \Delta \) is a constant.

(a) Find the state of the system at an arbitrary time \( t>0 \)

Note: This part of the problem does \textbf{not} deal with perturbation theory; find exact solution.

(b) Using the result of part (a), find the probabilities that after time \( t \) the system will end up in \( |m_{s1}, m_{s2}\rangle = |+, +\rangle, |-, -\rangle, |+, -\rangle \) and \( |-, +\rangle \) states.

(c) Now treat \( V \) as a perturbation (applied at \( t=0 \)) and calculate the probabilities of the transitions described in part (b) using first-order perturbation theory.

(d) Compare the exact solution found in part (b) with the approximate one found in part (c) and comment.
2. (15 pts) Consider elastic scattering of an electron by a nucleus so that the electron-nucleus interaction is treated as a perturbation and is described by some potential \( V(\mathbf{r}) \).

Use the Fermi’s golden rule to calculate transition rate from some initial state \( |i\rangle \) to some final state \( |f\rangle \). For this, you will need to calculate the matrix element of the perturbation potential and the density of states.

(a) Write down wavefunctions describing initial and final state of the electron.

Use plane waves as an approximation for the wave function of the electron.

(b) Write down the integral you need to solve to find the matrix element. Do you see any relation to a Fourier transform?

(c) Write down \( \rho(E) \)

(d) Combine results of (b) and (c) into the transition probability per unit time and differential cross-section. Discuss how one would probe this experimentally.

If we want to detect the scattered electron, does it matter where the detector is placed?

3. Reading assignment: Sakurai 5.5, 5.7, 5.8.