PH 585 AMO

Lecture # 24

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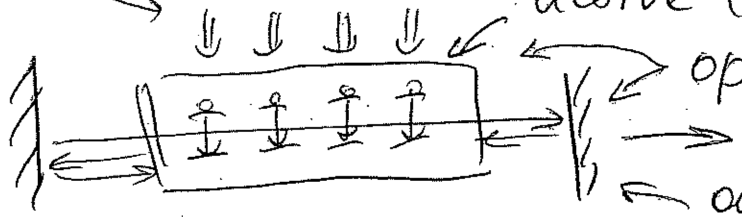
Lasers

Light amplification by stimulated emission of radiation

pumping

active (or gain) medium

optical cavity (resonator)



output coupler

$R \simeq 95\text{-}99\%$

$R = 100\%$

reflectance

transition rate for stimulated emission

Recall: $W_{21} = \frac{4\pi^2}{m^2 c} \frac{e^2}{4\pi\varepsilon_0} \frac{I(\omega_{21})}{\omega_{21}^2} |M_{21}(\omega_{21})|^2$

By the type of the active medium:

e.g.

- gas (He Ne) Ar^+, CO_2
- solid-state (Nd : YAG, Ruby, ...) Ti: sapphire
- excimer (Ar_2^*, KrF, XeCl, N_2, ...)
- semiconductor (GaAs, ...)
- dye (R6G, ...)
- free-electron

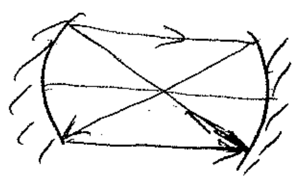
By pumping ⇒ – optically pumped (by flashlamps or laser diodes) ⇒ e.g. DPSS laser (diode pumped solid state) such as Nd : YVO_4 in my lab

– electrically pumped ⇒ i.e. by cw, rf, or pulsed current flowing in a conductive medium such as ionized gas or semiconductor

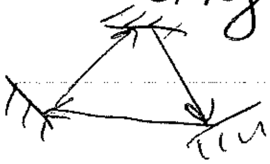
gas dynamic, chemical, e-beam → less common

By resonator ⇒ plane parallel (Fabri-Perot)

⇒ spherical



⇒ ring



cw ← continuous wave

By pulse length ⇒ ns, ps, fs pulsed

Q-switched, mode-locked ⇒ actively, passively

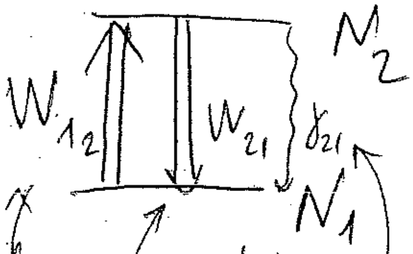
by mechanism of achieving pulses

Properties of laser radiation ⇒ monochromaticity

⇒ directionality

⇒ spatial & temporal coherence

Consider active medium $\Rightarrow$ start with 2-level system



N_2 ← number of atoms per unit volume in state 2

$N = N_1 + N_2$ ← total number of atoms

abs. ← W_{12}

stimulated em. ← W_{21}

γ_{21} ← rate for spontaneous emission and non-rad. decay

Then, $$\frac{dN_1}{dt} = -W_{12} N_1(t) + (W_{21} + \gamma_{21}) N_2(t)$$

Since $\frac{dN}{dt} = 0 \Rightarrow \frac{dN_1}{dt} = -\frac{dN_2}{dt}$

Population difference $\Delta N = N_1 - N_2$ ← for non-degenerate levels

Beam $\Rightarrow$ z

$$\frac{dp}{dt} = -N_1 \hbar\omega W_{21} + N_2 \hbar\omega W_{12} =$$

neglect γ_{21}

$\frac{dp}{dt}$ ← change of energy density; $\frac{\text{energy}}{\text{volume} \cdot \text{time}}$

$$= \underbrace{\hbar\omega W_{12}}_{= \sigma I}(N_2 - N_1) =$$

σ ← cross-section; I ← intensity

$$= \frac{dI}{dz}$$

I ← $\frac{\text{energy}}{\text{area time}}$ = intensity

So, $$\frac{dI}{dz} = \sigma I \underbrace{(N_2 - N_1)}_{= -\Delta N} \Rightarrow$$

if $N_2 > N_1 \Rightarrow$ amplification

$N_2 < N_1 \Rightarrow$ absorption

Back to rate equations $\Rightarrow$ add them up $\Rightarrow$ (4)

$$\frac{d}{dt}\overbrace{(N_1 - N_2)}^{=\Delta N(t)} = -\underbrace{(W_{12}+W_{21})}_{= 2W_{12}}\Delta N(t) + 2\gamma_{21}N_2(t)$$

$$= -2W_{12}\,\Delta N(t) + \gamma_{21}\underbrace{(N_2(t) + N - N_1(t))}_{= N - \Delta N(t)} \Rightarrow$$

$$\frac{d\,\Delta N(t)}{dt} = -2W_{12}\,\Delta N(t) - \gamma_{21}(\Delta N(t) - N)$$

$\gamma_{21} \equiv \frac{1}{T_1} \leftarrow$ population recovery time (or energy decay) (or longitudinal relaxation time) (or on-diagonal rel...

If $I = 0 \Rightarrow \frac{d\,\Delta N(t)}{dt} = \frac{N}{T_1} - \frac{\Delta N(t)}{T_1} \Rightarrow$

no pumping

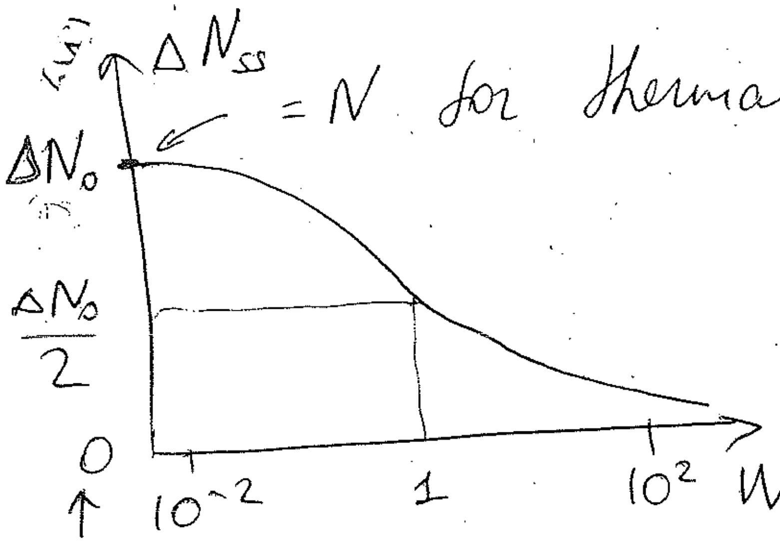
$$\Delta N(t) = N\left(1 - e^{-t/T_1}\right) + \Delta N_0\, e^{-t/T_1}$$

ΔN_0: diff. in population at $t=0$

Steady-state:

$$\frac{d\Delta N}{dt} = 0 \Rightarrow \Delta N_{ss} = \frac{N}{1 + 2W_{12}T_1} = \frac{N}{1 + W_{12}/W_{sat}}$$

$$W_{sat} \equiv \frac{1}{2T_1}$$



$= N$ for thermal equilibrium and optical transitions (i.e. $N_2(0) = N_1(0) e^{-\frac{\hbar\omega}{kT}}$, $N_2 \approx 0$)

$\Rightarrow$ leads to saturation of absorption and gain

$\boxed{N_1 = N_2 = \frac{N}{2} \text{ at best!}}$

$$\alpha = \frac{\alpha_0}{1 + I/I_{sat}} \Leftarrow$$

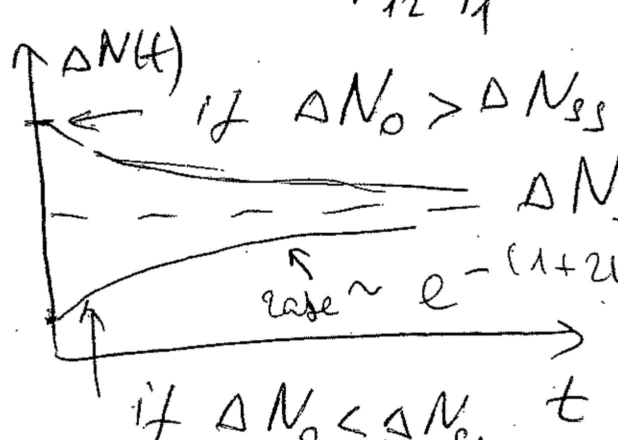
I: pumping intensity

gain $\sim \sigma(N_2 - N_1)$, absorption $\sim \sigma(N_1 - N_2)$

Time dependence of $\Delta N(t)$:

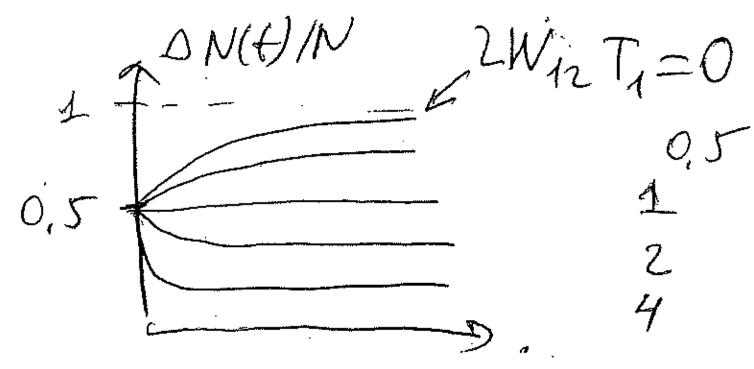
$$\Delta N(t) = \Delta N_{ss} + \left[\Delta N_0 - \Delta N_{ss}\right] e^{-\frac{(1+2W_{12}T_1)t}{T_1}}$$

$\Delta N_{ss} = \frac{N}{1+2W_{12}T_1}$, ΔN_0 at $t=0$

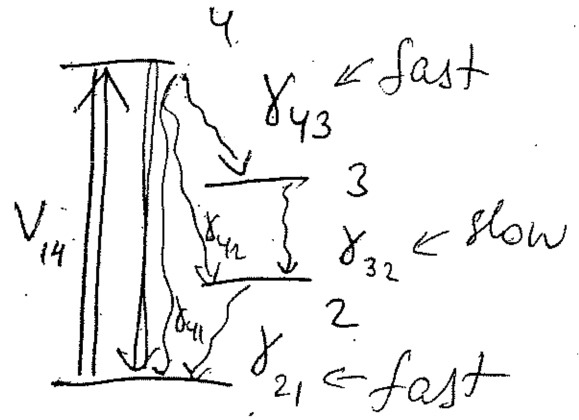


$\leftarrow$ faster as $I \uparrow$

if $\Delta N_0 = \frac{N}{2} \Rightarrow$



Four-level pumping analysis



$$W_{14} = W_{41} = W_p \leftarrow \text{pumping}$$

$$\frac{dN_4}{dt} = W_p (N_1 - N_4) - \underbrace{(\gamma_{43} + \gamma_{42} + \gamma_{41})}_{= \frac{1}{\tau_4}} N_4$$

$\frac{1}{\tau_4}$ ← lifetime of level 4

Steady-state

$$\frac{dN_4}{dt} = 0 \Rightarrow N_4 = \frac{W_p \tau_4}{1 + W_p \tau_4} N_1 \simeq W_p \tau_4 N_1$$

if $W_p \tau_4 \ll 1$ ← for most practical lasers

not pumped by W_p

$$\frac{dN_3}{dt} = \gamma_{43} N_4 - \underbrace{(\gamma_{32} + \gamma_{31})}_{\frac{1}{\tau_3}} N_3 = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3}$$

$$\frac{dN_2}{dt} = \gamma_{42} N_4 + \gamma_{32} N_3 - \gamma_{21} N_2 = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$

Steady-state $\Rightarrow$ $N_3 = \frac{\tau_3}{\tau_{43}} N_4 \Rightarrow$ if $\tau_3 \gg \tau_{43} \Rightarrow N_3 \gg N_4$

Also, $\frac{N_2}{\tau_{21}} = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} = \left(\frac{\tau_{43}}{\tau_3 \tau_{42}} + \frac{1}{\tau_{32}}\right) N_3 \Rightarrow$

$$N_2 = \underbrace{\beta}_{\parallel} N_3$$

$$\frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43}\tau_{21}}{\tau_{42}\tau_3} \Rightarrow \text{if } \beta < 1 \Rightarrow \underline{N_2 < N_3}$$

$\Downarrow$

inverted population

In a good laser medium

$\Downarrow$

$$\gamma_{42} \simeq 0 \quad (\tau_{42} = \infty) \Rightarrow \underline{\underline{\beta \simeq \frac{\tau_{21}}{\tau_{32}}}}$$

$\leftarrow$ need this short

$\leftarrow$ long

Another parameter of interest:

$$\eta = \frac{\gamma_{43}}{\gamma_4} \cdot \frac{\gamma_{rad}}{\gamma_3} = \underbrace{\frac{\tau_4}{\tau_{43}}} \cdot \underbrace{\frac{\tau_3}{\tau_{rad}}}$$

$\leftarrow$ radiative decay rate for $3 \to 2$ (laser transition)

$\uparrow$ fluorescent quantum efficiency

$\uparrow$ # fluo photons spontaneously emitted on the laser transition per # pump photons absorbed on the pump transition

$\uparrow$ fraction of excited atoms that end up in the level 3

$\leftarrow$ fraction of total decay of level 3 due to radiative processes

Now solve for $\Delta N = N_3 - N_2$ ← population inversion

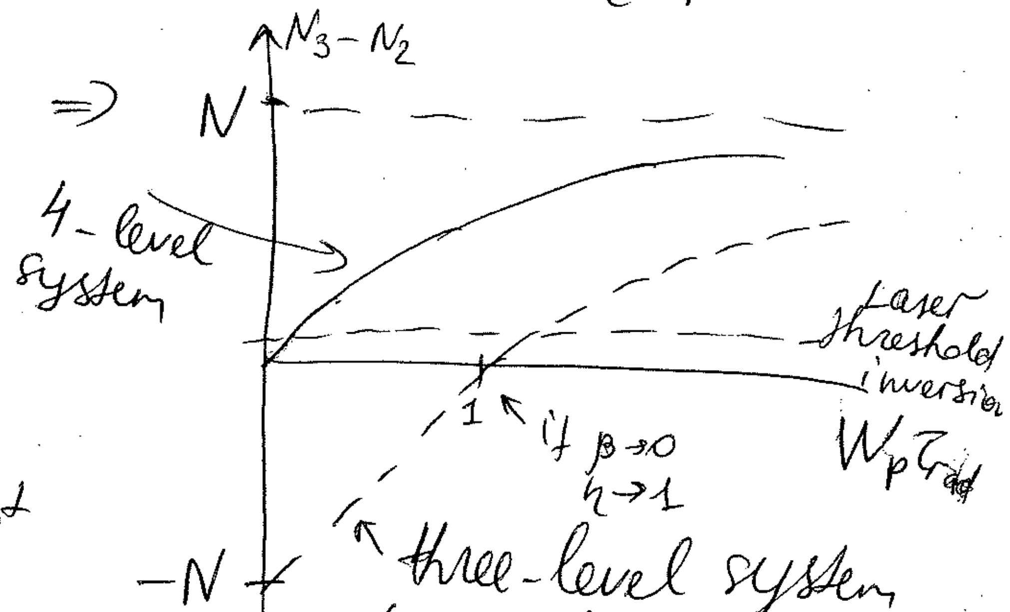
$$N_3 - N_2 = N_3(1-\beta) \overset{\text{show!}}{=} \frac{N(1-\beta)\eta W_p \tau_{rad}}{1 + [1+\beta+2\tau_{43}/\tau_{rad}]\eta W_p \tau_{rad}}$$

$$N_3 = \frac{\tau_3}{\tau_{43}} \underbrace{N_4}_{= \frac{W_p \tau_4}{1+W_p\tau_4} N_1}$$

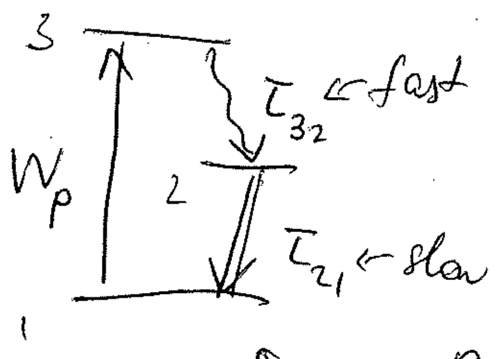
$N = N_1 + N_2 + N_3 + N_4$ ← conservation of atoms

If $\tau_{43} \ll \tau_{rad} \Rightarrow \frac{\Delta N}{N} = \frac{(1-\beta)\eta W_p \tau_{rad}}{1 + (1+\beta)\eta W_p \tau_{rad}} \approx$

$\underset{\beta \to 0}{\approx} \frac{W_p \tau_{rad}}{1 + W_p \tau_{rad}} \Rightarrow$



three-level system (need higher pumping power!)



← e.g. Ruby laser

$$\underbrace{\frac{\Delta N}{N}}_{= N_2 - N_1} = \frac{(1-\beta)\eta W_p \tau_{rad} - 1}{(1+2\beta)\eta W_p \tau_{rad} + 1}$$

Laser cavity modes

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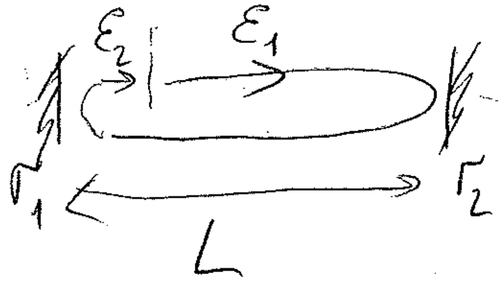
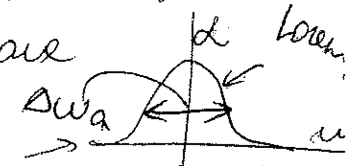


Steady-state: $\mathcal{E}_2 = \mathcal{E}_1$

electric field of EM wave

$$I(z) = I_0 e^{-\underbrace{2\alpha(\omega)}z}$$

absorption coefficient



$\alpha(\omega) \sim N_1 - N_2$

Laser amplification ⇒

$$I(z) = I_0 e^{2\alpha(\omega) z}$$

→ when $N_2 > N_1$

Steady-state: $$\mathcal{E}_2 = \mathcal{E}_1 r_2 r_1 e^{\underbrace{2\alpha L}_{\text{amplification}} - \underbrace{i \cdot \frac{2\omega L}{c}}_{\text{phase}}} = \mathcal{E}_1$$

in the absence of losses

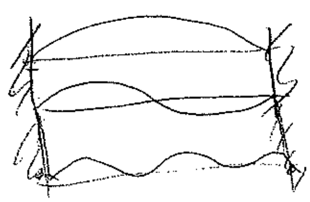
$$\alpha_{th} = \frac{1}{4L} \ln \frac{1}{R_1 R_2}$$ ← $R = |r|^2$

power reflectivity

$$e^{-i\frac{2\omega L}{c}} = e^{-2\pi q i} \Rightarrow \frac{2\omega L}{c} = 2\pi q \Rightarrow \omega = \omega_q = 2\pi q\left(\frac{c}{2L}\right)$$

integer

longitudinal modes



← various modes of the resonator

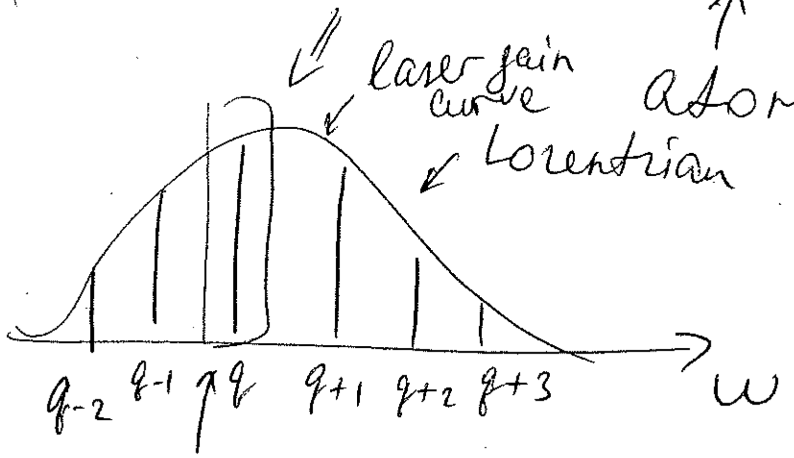
estimate: $q = \frac{\omega_q L}{\pi c} = \frac{2L}{\lambda_q} \approx 4 \cdot 10^6$ modes number

$L \sim 1\,m$, 500 nm

$\Delta\omega_q = \omega_{q+1} - \omega_q = \frac{\pi c}{L} \sim 10^9\ \cancel{GHz}\ s^{-1}$ ($L \sim 1\,m$)

$\Delta\nu_q = \frac{c}{2L} = 1.5 \cdot 10^8\ Hz =$

Typically, $\Delta\omega_q < \Delta\omega_a$ ← atomic linewidth



single-mode laser: select only one mode

e.g. using an additional resonator ← Fabri-Perot in the cavity