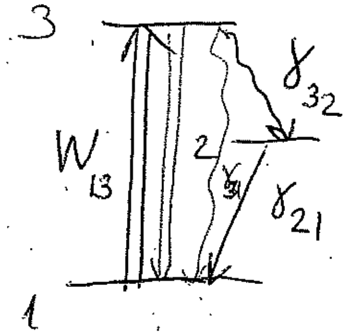
PH 585 AMO

Solution of HW #7

(1)

Problem #1 three-level system



$$W_{13} = W_{31} = W_p \;;\quad N = N_1 + N_2 + N_3$$

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - \gamma_{31} N_3 - \gamma_{32} N_3$$

$$\frac{dN_2}{dt} = \gamma_{32} N_3 - \gamma_{21} N_2$$

neglect direct pumping

Steady-state: $N_3 = \underbrace{\frac{\gamma_{21}}{\gamma_{32}}}_{=\beta} N_2$; $\overbrace{N_3}^{N - N_1 - N_2} = \frac{W_p}{\underbrace{\gamma_{31} + \gamma_{32}}_{=\gamma_3}}(N_1 - N_3) =$

$= \beta N_2$; introduce $\eta = \frac{\gamma_{32}}{\gamma_3} \cdot \frac{\gamma_{rad}}{\gamma_{21}} \Rightarrow \gamma_3 = \frac{1}{\beta\eta}\gamma_{rad} = \frac{1}{\eta\beta\,\tau_{rad}}$

$\gamma_3 \leftarrow \gamma_{31} + \gamma_{32}$

$$N_2(1+\beta) = N - N_1$$

$$\frac{W_p}{\gamma_3}(N_1 - N_3) = \frac{\beta}{1+\beta}(N - N_1)$$

$$\frac{W_p}{\gamma_3}\left(\frac{N_1}{N_2}-\beta\right)=\beta \Rightarrow \frac{N_1}{N_2}=\frac{\beta\overbrace{\gamma_3}^{=\frac{1}{\eta\tau_{rad}}}}{W_p}+\beta=\frac{1}{W_p\tau_{rad}\eta}+\beta$$

$$N = N_2\left(\frac{1}{W_p\tau_{rad}\eta}+\beta\right)+N_2+\beta N_2 = N_2\left(1+2\beta+\frac{1}{W_p\tau_{rad}\eta}\right)$$

$$N_2-N_1 = N_2-N_2\left(\frac{1}{W_p\tau_{rad}\eta}+\beta\right)=\frac{N}{1+2\beta+\frac{1}{W_p\tau_{rad}\eta}}\left[1-\frac{1}{W_p\tau_{rad}\eta}-\beta\right]$$

$$\frac{N_2-N_1}{N}=\frac{(1-\beta)W_p\tau_{rad}\eta-1}{(1+2\beta)W_p\tau_{rad}\eta+1}$$

Inversion occurs when $W_p\tau_{rad} > \dfrac{1}{(1-\beta)\eta}$