PH585
AMO

Solution of HW #3

Problem #1

$$\Delta E_1 = -E_n \frac{(Z\alpha)^2}{n^2}\left[\frac{3}{4} - \frac{n}{l+1/2}\right]$$

$$\Delta E_2 = -E_n \frac{(Z\alpha)^2}{2nl(l+1/2)(l+1)} \times \begin{cases} l & \text{if } j = l+1/2 \\ -l-1 & \text{if } j = l-1/2 \end{cases}$$

; 0 if $l=0$

$$\Delta E_3 = -E_n \frac{(Z\alpha)^2}{n} \text{ for } l=0 \quad (0 \text{ for } l \neq 0)$$

$\underline{n=2} \Rightarrow$ $2S \Rightarrow l=0,\ S=1/2 \Rightarrow j=1/2$

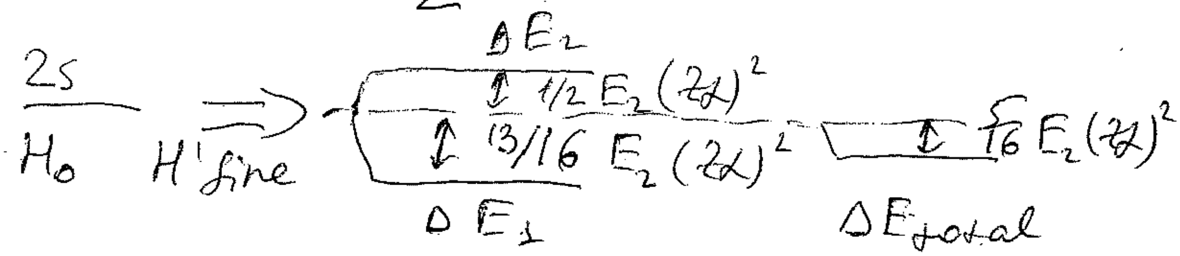
$2P \Rightarrow l=1,\ S=1/2 \Rightarrow j=\frac{1}{2}, \frac{3}{2}$

Then, for $\underline{2S}$:

$$\Delta E_1 = -E_2 \frac{(Z\alpha)^2}{4}\left[\frac{3}{4} - \frac{2}{1/2}\right] = E_2 \frac{(Z\alpha)^2}{4} \cdot \frac{13}{4} = \frac{13}{16} E_2 (Z\alpha)^2$$

$$\Delta E_2 = 0$$

$$\Delta E_3 = -E_2 \frac{(Z\alpha)^2}{2}$$



$$E_2 = -\frac{E_I}{4}$$

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$\Rightarrow$ 0 if $l=0$

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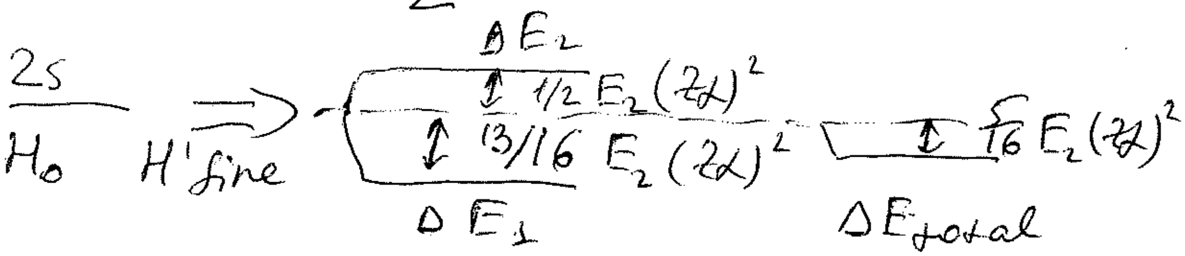
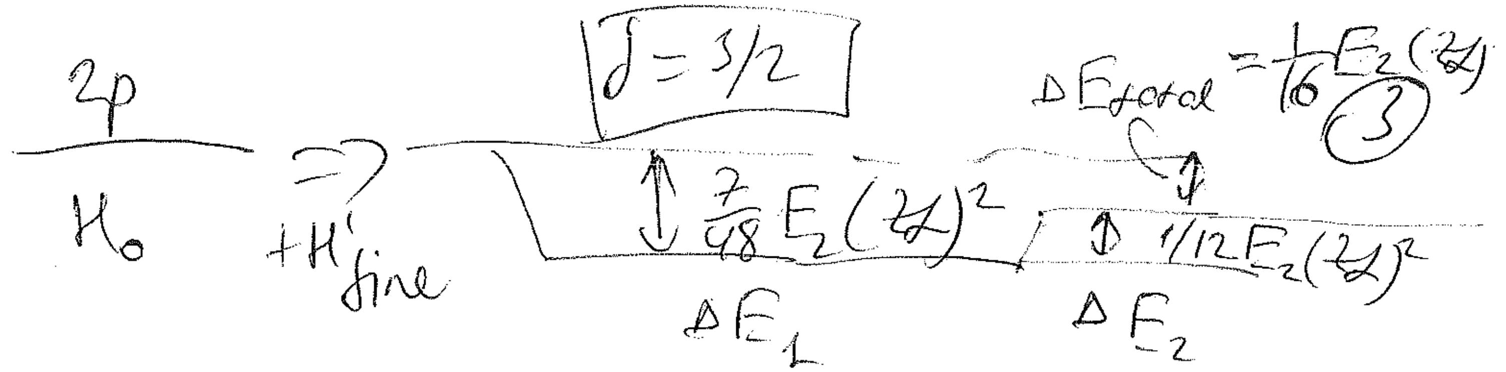
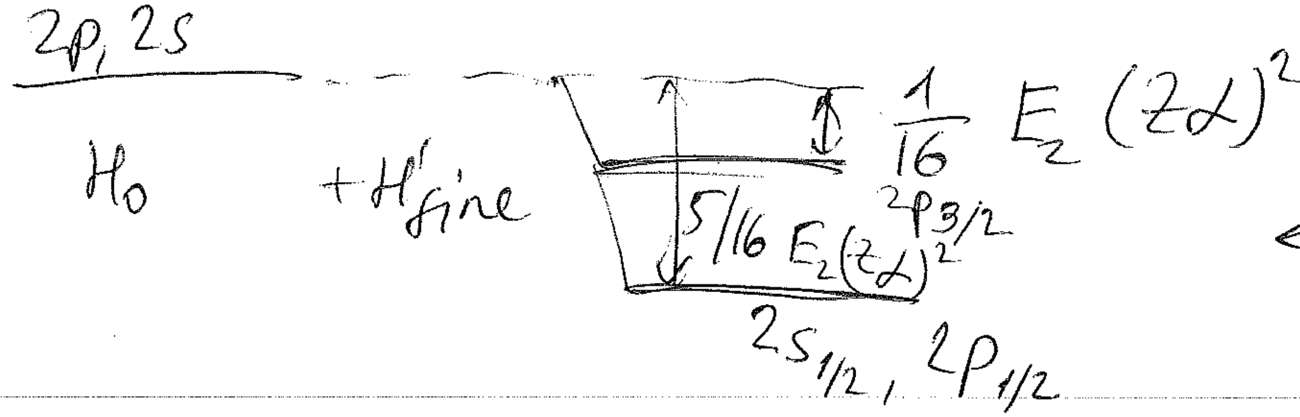


$$E_2 = -\frac{E_I}{4}$$



Everything together:



Problem #2 Fine structure of H_α

H_α: $n=3 \to n=2 \to 2S_{1/2}, {}^2P_{1/2}$; $2P_{3/2}$ states

$\Downarrow ?$

$3s, 3p, 3d \Rightarrow$ find ΔE_{total}

$$\Delta E_{total} = E_n \frac{(Z\alpha)^2}{n^2} \left(\frac{n}{j+1/2} - \frac{3}{4} \right)$$

(5,28)

$3s \Rightarrow l = 0,\ s = 1/2 \Rightarrow j = 1/2$

$3p \Rightarrow l = 1,\ s = 1/2 \Rightarrow j = 1/2, 3/2$

$3d \Rightarrow l = 2,\ s = 1/2 \Rightarrow j = 3/2, 5/2$

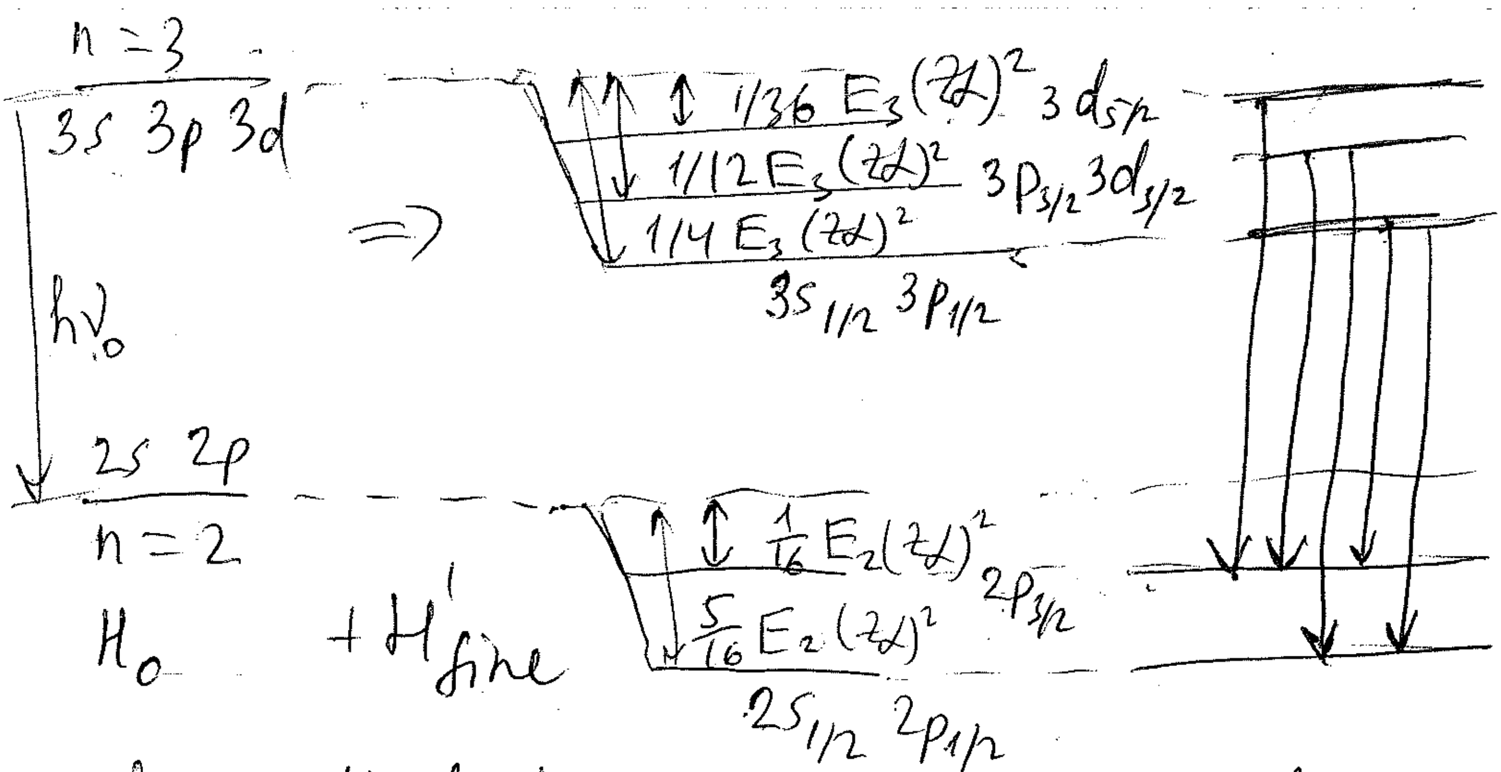
So,

$$\Delta E_{total} \underset{j=1/2}{=} E_3 \frac{(Z\alpha)^2}{9}\left(\frac{3}{1/2+1/2}-\frac{3}{4}\right) = \frac{1}{4}E_3 (Z\alpha)^2$$

$$\Delta E_{total} \underset{j=3/2}{=} E_3 \frac{(Z\alpha)^2}{9}\left(\frac{3}{\frac{3}{2}+\frac{1}{2}}-\frac{3}{4}\right) = \frac{1}{12}E_3 (Z\alpha)^2$$

$$\Delta E_{total} \underset{j=5/2}{=} E_3 \frac{(Z\alpha)^2}{9}\left(\frac{3}{5/2+1/2}-\frac{3}{4}\right) = \frac{1}{36}E_3 (Z\alpha)^2$$

Then,



electric dipole transitions $\Rightarrow \Delta l = \pm 1,\ \Delta j = 0, \pm 1$

So, overall 7 transitions:

$3d_{5/2} \to 2p_{3/2}$

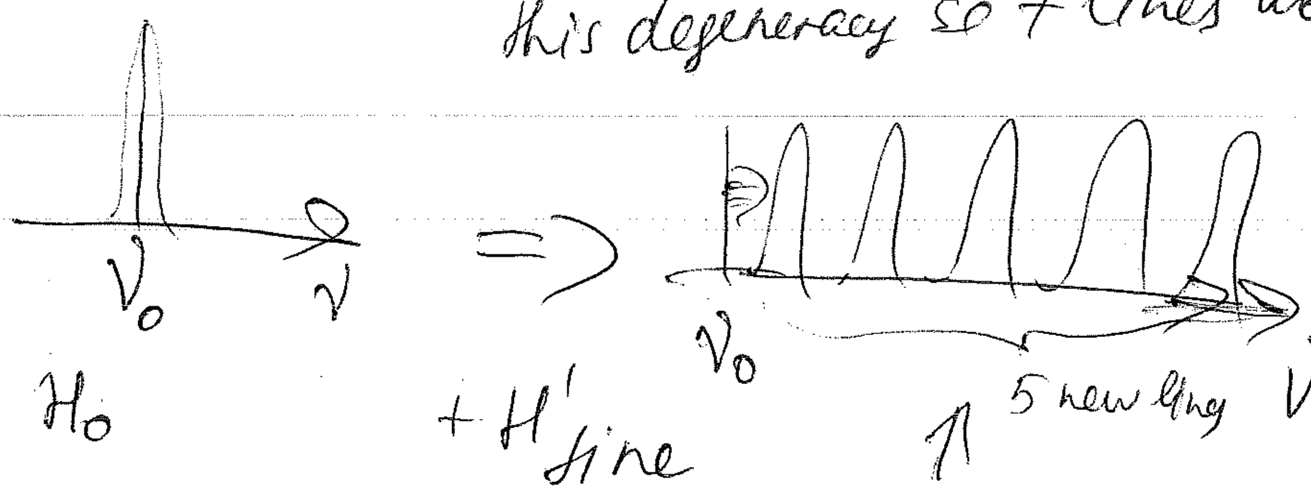
$\left.\begin{array}{l} 3p_{3/2} \to 2s_{1/2} \\ 3d_{3/2} \to 2p_{3/2} \\ 3d_{3/2} \to 2p_{1/2} \end{array}\right]$ same ν

$\left.\begin{array}{l} 3s_{1/2} \to 2p_{1/2} \\ 3p_{1/2} \to 2s_{1/2} \end{array}\right)$ same ν

$3s_{1/2} \to 2p_{3/2}$

But only 5 distinct frequencies $\Rightarrow$

(note: Lamb shift would remove this degeneracy so 7 lines would be visible)



see Fig. 5.5 for actual line intensities

Problem #3

$$\omega = \omega_0 \left(1 \mp \frac{v}{c}\right) \Rightarrow \frac{\omega}{\omega_0} - 1 = \mp \frac{v}{c} \Rightarrow$$

$$v^2 = c^2 \left(\frac{\omega}{\omega_0} - 1\right)^2 \Rightarrow dN \underset{(4.189)}{=} N_0 e^{-\frac{Mv^2}{2k_BT}} dv =$$

$$= N_0 e^{-\frac{Mc^2}{2k_BT}\left(\frac{\omega}{\omega_0}-1\right)^2} \left(\mp \frac{c}{\omega_0}\right) d\omega$$

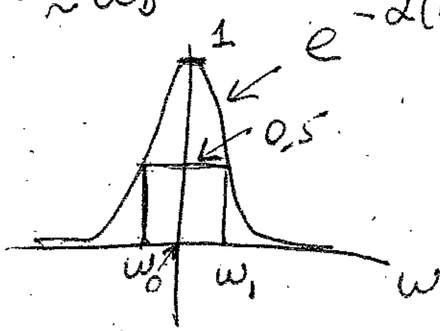
$$I(\omega) \underset{(4.13)}{=} \frac{\hbar\omega N(\omega) c}{V} = \frac{\hbar\omega}{V} \frac{c}{\omega_0} N_0 e^{-\frac{Mc^2}{2k_BT}\left(\frac{\omega}{\omega_0}-1\right)^2} =$$

$$N(\omega) = \left|\frac{dN}{d\omega}\right|$$

$$I(\omega) \sim \left|\frac{dN}{dv}\right|$$

$$\underset{\omega \approx \omega_0}{=} \frac{\hbar c N_0}{V} e^{-\frac{Mc^2}{2k_BT}\left(\frac{\omega-\omega_0}{\omega_0}\right)^2}$$

$$e^{-\alpha(\omega-\omega_0)^2}, \quad \alpha = \frac{Mc^2}{2k_BT\omega_0^2}$$



$$e^{-\alpha(\omega_1-\omega_0)^2} = \frac{1}{2} \Rightarrow$$

$$\alpha(\omega_1-\omega_0)^2 = \ln 2$$

(4.191)

$$(\omega_1-\omega_0)^2 = \frac{\ln 2}{\alpha} = \frac{2k_BT\omega_0^2}{Mc^2}\ln 2$$

so total width

$$\Delta\omega^D = 2\cdot(\omega_1-\omega_0) = \frac{2\omega_0}{c}\sqrt{\frac{2k_BT\ln 2}{M}}$$