PH 585 AMO Solutions of Homework #1

Problem #1 BsJ 3,4

$^3H \Rightarrow {}^3He$

$\Uparrow$ triton $\Uparrow$

$\underbrace{1p + 2n\,(+1e)}_{\text{tritium}}$ $2p + 1n\,(+1e)$

$\Downarrow$ $\downarrow$ He^+

$Z=1$ $Z=2$

$$\underset{(\text{triton})}{\Psi_{1s}} = \frac{1}{\sqrt{\pi}} \frac{1}{a_0^{3/2}} e^{-\frac{r}{a_0}} \quad ; \quad \underset{(He^+)}{\Psi_{1s}} = \frac{1}{\sqrt{\pi}} \left(\frac{2}{a_0}\right)^{3/2} e^{-\frac{2r}{a_0}}$$

$a_\mu \approx a_0$; $2 = Z$

(a) $$\underset{\substack{1s \to 1s \\ T \;\; He^+}}{\mathcal{P}} = \left| \langle \underset{\text{triton}}{\Psi_{1s}} | \underset{He^+}{\Psi_{1s}} \rangle \right|^2 = \left| \frac{1}{\pi} \frac{2^{3/2}}{a_0^3} \underbrace{\int_0^\infty e^{-\frac{3r}{a_0}} r^2 dr}_{\left(\frac{a_0}{3}\right)^3 \underset{2!}{\Gamma(3)}} \cdot 4\pi \right|$$

$$= \left| \frac{2^{5/2} \cdot 4}{3^3} \right|^2 = \frac{2^9}{3^6} \simeq 0{,}7$$

(b) $$1 - \underset{\substack{1s \to 1s \\ T \;\; He^+}}{\mathcal{P}} = 0{,}3$$

(c) $$\underset{He^+}{\Psi_{2s}} = \frac{1}{2\sqrt{2\pi}} \left(\frac{2}{a_0}\right)^{3/2} \left(1 - \frac{2r}{2a_0}\right) e^{-\frac{r}{a_0}}$$

$$\mathcal{P}_{\substack{1s\to 2s\\ T\quad He^+}} = \left|\langle \Psi_{\substack{1s\\T}} | \Psi_{\substack{2s\\He^+}} \rangle\right|^2 = \left| \cancel{\frac{1}{\pi}}\, \cancel{\frac{2^{3/2}}{a_0^3}}\, \cancel{\frac{1}{2\sqrt{2}}} \cdot \cancel{4\pi} \cdot \right.$$

$$\left. \cdot \underbrace{\int_0^\infty e^{-\frac{2r}{a_0}}\left(1-\frac{r}{a_0}\right) r^2 dr}_{} \right|^2 = \left| \frac{4}{a_0^3}\cdot\left(\frac{a_0^3}{8}\cdot 2 - \frac{a_0^3}{16}\cdot 6\right)\right|^2$$

$$\| \quad \left(\frac{a_0}{2}\right)^3 \underbrace{\Gamma(3)}_{2!} - \frac{1}{a_0}\cdot\left(\frac{a_0}{2}\right)^4 \underbrace{\Gamma(4)}_{3!}$$

$$= \left| 1 - \frac{3}{2} \right|^2 = \underline{0{,}25}$$

1) $l \neq 0 \Rightarrow \Psi_{\substack{nlm\\He^+}} = R_{nl}(r)\, Y_{lm}(\theta,\varphi) \Rightarrow$

$(l \neq 0,\ |m| \leq l)$

$$\mathcal{P} = \left|\langle \Psi_{\substack{1s\\T}} | \Psi_{\substack{nlm\\He^+}} \rangle\right|^2 = \left| \int_0^\infty \underbrace{R_{10}(r)}_{T}\, \underbrace{R_{nl}(r)}_{He^+}\, r^2 dr \right. \cdot$$

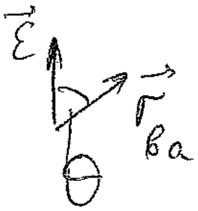
$$\left. \cdot \int \underbrace{Y_{00}}_{T\,(1s)}\, \underbrace{Y_{lm}}_{He^+\,(p,d,\dots)}\, d\Omega \right|^2 = 0$$

states other than s

orthogonality of spherical harmonics

Problem # 2 from HW #1   B & J 4.2

(a) $$W_{ba}^{D} = \frac{4\pi^2}{c\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right) I(\omega_{ba})|\vec{r}_{ba}|^2\cos^2\theta$$



If θ is random $\Rightarrow$ $$\overline{W}_{ba}^{D} = \frac{4\pi^2}{c\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right) \cdot I(\omega_{ba})\,|\vec{r}_{ba}|^2\,\frac{1}{4\pi}\underbrace{\int \cos^2\theta\, d\Omega}_{2\pi\int_0^\pi \cos^2\theta\,\sin\theta\, d\theta \,=\, \frac{4\pi}{3}} = \frac{4\pi^2}{3c\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right) I(\omega_{ba})\cdot|\vec{r}_{ba}|^2$$

(b) $$W_{ab}^{s,D} = \frac{1}{2\pi\hbar c^3}\,\frac{1}{4\pi\varepsilon_0}\,\omega_{ba}^3\int d\Omega\left\{\underbrace{|\vec{\varepsilon}_1\cdot\vec{D}_{ba}|^2}_{|\vec{D}_{ba}|^2\cos^2\theta_1} + \underbrace{|\vec{\varepsilon}_2\cdot\vec{D}_{ba}|^2}_{=\,|\vec{D}_{ba}|^2\cos^2\theta_2}\right\} = \frac{1}{3\pi\hbar c^3}\,\omega_{ba}^3\,|\vec{D}_{ba}|^2$$

$$\int d\Omega\left(\cos^2\theta_1 + \cos^2\theta_2\right) = \frac{8\pi}{3}$$

↑ all possible choices of θ_1 & $\theta_2 \in [0,\pi]$

Problem #3 Show $\bar{M}_{ab} = -M_{ba}^*$

$$\bar{M}_{ab} = \langle \psi_a | e^{-i\vec{k}\cdot\vec{r}} \vec{\varepsilon}\cdot\vec{\nabla} | \psi_b \rangle = \int \psi_a^* e^{-i\vec{k}\cdot\vec{r}} \vec{\varepsilon}\cdot\nabla\psi_b d\vec{r}$$

$$= \vec{\varepsilon}\cdot \underbrace{\int \vec{\nabla}\left(\psi_a^* e^{-i\vec{k}\cdot\vec{r}} \psi_b\right) d\vec{r}}_{=0 \ (\psi_{a,b}\to 0 \text{ at } |\vec{r}|\to\infty)} - \int \vec{\varepsilon}\cdot\vec{\nabla}\psi_a^* e^{-i\vec{k}\cdot\vec{r}} \psi_b \, d\vec{r} =$$

$$\vec{\varepsilon}\cdot\vec{\nabla}\left(\psi_a^* e^{-i\vec{k}\cdot\vec{r}} \psi_b\right) = \vec{\varepsilon}\cdot\vec{\nabla}\psi_a^* e^{-i\vec{k}\cdot\vec{r}} \psi_b + \underbrace{\vec{\varepsilon}\cdot\psi_a^*(-i\vec{k}) e^{-i\vec{k}\cdot\vec{r}} \cdot \psi_b}_{=0 \ \Leftarrow \ \vec{\varepsilon}\cdot\vec{k}=0} + \underbrace{\vec{\varepsilon}\cdot\psi_a^* e^{-i\vec{k}\cdot\vec{r}} \vec{\nabla}\psi_b}$$

$$= -\int \psi_b e^{-i\vec{k}\cdot\vec{r}} \vec{\varepsilon}\cdot\nabla\psi_a^* d\vec{r} \underset{\substack{\text{assume} \\ \vec{\varepsilon} \text{ real}}}{=} -\Big[\underbrace{\int \psi_b^* e^{i\vec{k}\cdot\vec{r}} \vec{\varepsilon}\cdot\vec{\nabla}\psi_a d\vec{r}}_{= M_{ba} \ \leftarrow \ \text{Eq. (4.40) in B\&J}}\Big]^*$$

$$= -M_{ba}^*$$

Problem #4

Show $\boxed{\vec{P}_{ba} = i m \omega_{ba} \vec{r}_{ba}}$ $\qquad \vec{p} = m\dot{\vec{r}}$

$$\dot{\vec{r}} = \frac{1}{i\hbar}[\vec{r}, H_0] \Rightarrow \langle \Psi_b | \dot{\vec{r}} | \Psi_a \rangle = \frac{1}{i\hbar} \langle \Psi_b | \vec{r} H_0 - H_0 \vec{r} | \Psi_a \rangle$$

$E_b \langle \Psi_b |$ $\qquad E_a \Psi_a$

Heisenberg equation of motion

$$= -\frac{1}{i\hbar} \underbrace{(E_b - E_a)}_{\omega_{ba}} \underbrace{\langle \Psi_b | \vec{r} | \Psi_a \rangle}_{\vec{r}_{ba}}$$