

Worksheet #1

Wednesday, September 23, 2026

Name:

Questions:

The state of the system at time $t = 0$ is given by:

$$|\psi\rangle = \frac{1}{3}|\varphi_1\rangle + \frac{1}{6}|\varphi_2\rangle + \frac{2}{3}|\varphi_3\rangle,$$

where $|\varphi_n\rangle$ states are eigenstates of the system's Hamiltonian: $H|\varphi_n\rangle = n\varepsilon_0|\varphi_n\rangle$.

(a) If we measure energy, what values do we get and with what probabilities?

possible energy values: $E_n = n\varepsilon_0 \Rightarrow$

$$\begin{array}{l} \varepsilon_0 \\ 2\varepsilon_0 \\ 3\varepsilon_0 \end{array}$$

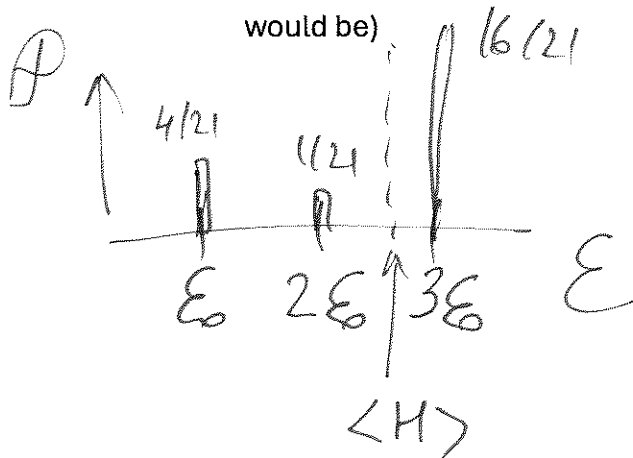
$$\text{probabilities: } P_n(n\varepsilon_0) = \frac{|\langle\varphi_n|\psi\rangle|^2}{\langle\psi|\psi\rangle}$$

$$\langle\psi|\psi\rangle = \frac{1}{9} + \frac{1}{36} + \frac{4}{9} = \frac{5}{9} + \frac{1}{36} = \frac{21}{36} = \frac{7}{12}$$

$$P_{n=1} = \frac{1/9}{7/12} = \frac{1}{9} \cdot \frac{12}{7} = \frac{4}{21} ; P_{n=2} = \frac{1/36}{7/12} = \frac{1}{36} \cdot \frac{12}{7} = \frac{1}{21} ; P_{n=3} = \frac{4/9}{7/12} = \frac{16}{21}$$

$$\text{Check: } \frac{4}{21} + \frac{1}{21} + \frac{16}{21} = 1$$

(b) Based on the results of part (a), predict where the average energy would be on the energy scale (plot the probability histogram and show where the average energy would be)



(c) **If you have time:** what is the average energy? Does it follow the prediction you made in (b)?

$$\begin{aligned}
 \langle H \rangle &= \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = P_{n=1} \cdot \epsilon_0 + P_{n=2} \cdot 2\epsilon_0 + \\
 &+ P_{n=3} \cdot 3\epsilon_0 = \frac{4}{21} \cdot \epsilon_0 + \frac{1}{21} \cdot 2\epsilon_0 + \frac{16}{21} \cdot 3\epsilon_0 = \\
 &= \frac{54}{21} \epsilon_0 = \frac{18}{7} \epsilon_0 \approx 2.6 \epsilon_0
 \end{aligned}$$

↑ matches prediction of (b)!

(d) **If you have time:** what is the state of the system at a later time t ?

$$\begin{aligned}
 |\psi(t)\rangle &= \left(\frac{1}{3} e^{-\frac{i}{\hbar} \epsilon_0 t} |\psi_1\rangle + \frac{1}{6} e^{-\frac{2i}{\hbar} \epsilon_0 t} |\psi_2\rangle \right. \\
 &\left. + \frac{2}{3} e^{-\frac{3i}{\hbar} \epsilon_0 t} |\psi_3\rangle \right) \cdot \sqrt{\frac{12}{7}}
 \end{aligned}$$

↑
to normalise!