MTH 655, Winter 2011, Assignment 4

1. Apply the a-posteriori error estimators for B) H_0^1 norm to your implementation from Assgn.2, with $u(x) = \frac{1}{100}x(1-x)e^{10x^2}$. Use uniform and nonuniform grid: suggest a way, guided by the estimator, to adapt the grid keeping number of nodes fixed so that the error is i) below a certain tolerance $\tau = 10^{-2}, 10^{-3}$, or/and ii) more or less evenly distributed over elements. Comment on advantages using nonuniform grid.

A) Do B) and use estimators for the i) error in L^2, ii) pointwise error in $u(.9)$.

2. Write out details of FE formulation for the problem $-u'' + cu = f$ on $(0, 2)$, with $u'(0) = 0, u(2) = 0$. Include the setting of the BVP, and computation of continuity and coercivity constants. Discuss the form of the matrix of the problem if exact integration/numerical integration is used. Implement it and verify that the method converges with the expected order for $u(x) = cos(\pi x/4)$. Adjust c so that i) $f \not\equiv 0$, ii) $f \equiv 0$.

3. Solve the eigenvalue problem using FE for the operator $Lu = -u''$, with boundary conditions as in 2. Check convergence of first two eigenvalues (show the corresponding eigenfunctions). Extra: use a-posteriori error estimator for $\lambda - \lambda_h$ derived in class.

4. Solve using collocation or Galerkin method the integral equation $\lambda u(x) - \int_0^1 e^{x-y}u(y)dy = f(x),\ 0 \le x \le 1$. Confirm the theoretical convergence of the method for $u(x) = e^x sin(\pi x)$. Use i) $\lambda = 10$, ii) $\lambda = 10^3$, and compare the errors for these two.

Comment on all the assumptions necessary for the theoretical estimates of convergence (compactness etc., smoothness of the solution) to hold.

Extra: use both methods, compare, and comment.