MTH 654/659 Fall 2011. Assignment 3 (lab 2)

Instructions: Please follow the instructions below and try to get through as much as possible during the lab. Ask me questions if you get stuck or need help.

Please write a (concise) lab report on your solution to Problem 1, 2b-c. You can start working on 3 for the purposes of your next regular assignment. [The assignments are designed in MATLAB but if you prefer a different language/environment, go ahead.]

Problems

1. Get familiar with Gaussian integration over triangles using the provided functions `tri_quadcofs` and `myint2d`. In particular, choose the appropriate quadrature order for the function you are integrating (see `myfun` in `myint2d`). Apply it to the integrals $\iint_T f(x,y)dA$ for each of $f_1(x,y) = 1$, $f_2(x,y) = x.^2 + y.^2$, $f_3(x,y) = x.^6$, $f_4(x,y) = sin(\pi x)y$ and to compute $\parallel f \parallel_{H^m(T)}$ for $m = 0, 1$ (check !). [Use only the appropriate order (which order do you need for optimal accuracy in each example ?) to decrease the time of computations].
2. Recall the interpolation estimates for $|v - I_h v|_{H^m(\Omega)} \leq Ch^{t-m}|v|_{H^t(\Omega)}$ where $0 \leq m \leq t$ proved in class. Recall that the constant depends on the mesh quality i.e. the ratio $\frac{r_2}{\rho_2}$ as shown in class. First, a) verify that the function is doing what it should on a simple two-triangle grid. Next, verify the order of convergence and dependence on mesh quality experimentally with $V_h = M_0^1(\mathcal{T}_h)$ and with b) a uniform mesh $\mathcal{T}_h$, c) a distorted mesh $\mathcal{T}_h$. You can use the provided function `interp2d`.
   To call $interp2(x, y, t)$, you need a mesh, e.g., created as follows (x,y are vectors of coordinates and t the vertices for each triangle). Some examples:
   ```
   %%% Example as in class: two triangles
   x = [0 1 0 1];
   y = [0 0 1 1];
   t = [1 2 3; 4 2 3];
   interp2(x,y,t)
   ```
   Or, you can use
   ```
   %%% Uniform grid
   nx = 10; ny = 10;
   xx = linspace(0,1,nx+1);
   yy = linspace(0,1,ny+1);
   [x,y] = meshgrid(xx,yy);
   t = delaunay(x,y);
   interp2(x,y,t)
   ```
   To solve b), you need to use a sequence of meshes i.e. varying nx, ny. To solve, c) you should use a distorted mesh (create one yourself $xx = g(xx)$ or ask for suggestions: any nonlinear function g from $(0, 10)$ into itself will do. The higher the derivative of g', the better).
3. Start preparing the grid for the domain $\Omega :=$ "$YourPersonalLetter$" (get it from me at the beginning of class). Note: you will likely not be able to use Delaunay triangulations unless Your Personal Letter is convex.