MTH 654/659 Fall 2011. Assignment 1

Instructions: Please solve 2 problems or more for extra credit. Preferred selection: solve at least one of problems 1-3 and at least one of 4-5.

Show all the relevant work. Group work is allowed on 4-5 but you have to write/type all your solutions yourself. The same concerns any (new pieces of) code you implemented.

In write-up, you are welcome to use LaTeX; see class website for the file(s).

In computational assignments, you are welcome to use the code `fem1d.m` posted at `http://www.math.oregonstate.edu/~mpesz/code/cmatlab/mth655/fem1d.m`
When reporting on your computational experiments, provide a concise summary of what you have observed/determined. [Do not just staple a bunch of graphs and papers together]. Convergence of a method is best shown by plotting the error in function of h using a log-log plot, or by computing the order of convergence, as dicussed in class.

Problems

1. Consider $f(x) = max(x^2, \sqrt{x})$ and compute its weak derivative for $x > 0$. To which of the spaces $C^k(\Omega), H^m(\Omega), W^{m,p}(\Omega)$ does f belong on $\Omega = (\frac{1}{4}, 2)$?

2. Work out details of the example $f(x, y) = loglog\frac{2}{r}$ (eq.1.8/p31 in text).

3. To which spaces $C^k, L^p, H^m, W^{m,p}$ does $g(r) = |r|^\beta$ belong on unit disk: $\{|r| < 1\} \subset \mathbb{R}^d$ in $d = 1, d = 2$? Consider in particular $\beta = 1, 2, -1, 1/2, -1/2$.

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4. Implement FE to solve the problem $-u'' = f(x), u(0) = u(1) = 0$ where f is chosen so that $u(x) = x(1-x)sin(x)$.
   Compute the approximate solution and error for different values of h in H^1 and L^2 norms (use numerical integration). Compare with the discrete norm
   $$\max_i |u(x_i) - u_h(x_i)|.$$
   What you see is the phenomenon of *superconvergence.*

5. Consider the problem $-au'' + cu = f$ with homogeneous Dirichlet boundary conditions, where $a > 0, c \geq 0$. Implement FE solution for this problem using linear elements. Use $a = \epsilon, c = 1, f \equiv 1$ with $\epsilon = 1, 1e^{-1}, 1e^{-2}$ etc.. Derive the analytical solution and test for convergence in H^1, L^2. What h do you have to use for small ϵ to get a decent behaving numerical solution ?
   What you see is an example of *a singularly perturbed problem.*