

MTH 622/Peszynska/Winter 2012, Assignment 2

Please show all your work. Use proper mathematical notation.

Turn in 1) for credit. All others can be used for extra credit.

1. Carry out calculations and the analysis of solving the analogue of Poisson's equation in 1D on an interval $-a < x < a$ for $a > 0$, with Dirichlet boundary data, using a fundamental solution and Green's function. In the process you will have calculated the Green's function explicitly. Pay attention to the singularities which may be of a slightly different nature than in $d = 2, 3$.
Hint: We discussed already the Green's function on $(0, 1)$ in MTH 621. The radially symmetric solution on $(-a, a)$ depends on $r = |x|$.
2. Carry out details necessary to prove the integral representation in 2D (we did this in class for 3D).
3. Show $G(x, y) = G(y, x)$.
4. Practice details calculating $\frac{\partial s}{\partial \eta}$, its integral, and other integrals on the boundary of $S_\xi(\epsilon)$ that we needed to derive the integral representation in 3D. In particular, work out details leading to [Glee, equation 8-3-8].
5. Carry out the calculations necessary to solve the Neumann BVP for Poisson's equation. Is the solution unique ?