MTH 622/Peszynska/Winter 2012, Assignment 1

Please show all your work. Use proper mathematical notation. Turn in 2), 3) for credit. 5) can be used for extra credit. Use 1), 4) as an exercise.

1. Verify whether the following functionals on $C^1([0,1])$ is linear
   i) $J(y) = \int_0^1 yy'dx$,
   ii) $J(y) = \int_0^1 \int_0^1 K(x,t)y(x)y(t)dxdt$,
   iii) $J(y) = \int_0^1 ysin(x)dx$,
   iv) $J(y) = y'(1/2) + y(0)$.
   v) $J(y) = f(y(0))$, where f is a given function.

2. Find, if possible, the solution to

$$a) \quad \inf_{u \in V} \int_0^1 x^2(u'(x))^2 dx. \; V = \{v \in C^1(0,1) : v(0) = 0, v(1) = 1\}$$

$$b) \quad \inf_{u \in V} \int_1^2 x^2(u'(x))^2 dx. \; V = \{v \in C^1(1,2) : v(1) = 0, v(2) = 1\}$$

   Interpret your findings properly.

3. Solve Laplace equation on a region between two circles of radii r_A, r_B, with $r_A < r_B$. Use boundary conditions $u|_{r=r_A} = A$, $u|_{r=r_B} = B$, where $A, B \in \mathbb{R}$.

4. Provide details of finding the fundamental solution to Laplace equation in $\mathbb{R}^3$.

5. Solve Laplace equation on a circle and rectangle using the method of separation of variables.
   On the rectangle $(0,1)^2$, use homogeneous boundary conditions everywhere except $u(x,0) = sin(\pi x), x \in [0,1]$.
   On the circle (radius $a = 1$) use boundary conditions $u(r,\theta) \mid_{r=a} = 1 - |\theta|/\pi$.
   Note: this solution can be looked up in many sources so if you want to turn it in for credit, please supply some value-added information such as plots of the first few terms of the Fourier series, discussion of convergence etc..