MTH 621/Peszynska/Fall 2011, Assignment 4

Please show all your work. Use proper mathematical notation. Solve 1-3 and one of 4, 5, 6 (or more for extra credit).

1. Solve 3-1.13

2. Consider $f(x) = x$, $g(x) = |x|$, and $h(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}$, each on $(-\pi, \pi)$.
   For each of these functions, consider what you would expect from the convergence of its Fourier series. Include the considerations of mean-square convergence, and pointwise and uniform convergence of the series combined with the discussion of the (lack of) continuity of the function and its derivatives. Use either all the theory from [Glee] or from the class handout.

3. For functions in the previous problem, compute their Fourier series and revisit the convergence issues.
   For computations, you can either a) use your results from Problem 1, or b) use Euler formulas directly. BE CONCISE.

4. Solve 3-2.2b). Use the series that you found to show

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

5. Consider the Hilbert space V and its subspace K. Find the Fourier coefficients for the best approximation in K of a given v.
   i) Let $K = \{v(x) = \alpha x^2; \alpha \in \mathbb{R}\}$ for $v(x) = x$ and $V = L^2(-1, 1)$.
   ii) Let $K = \{v(x) = \alpha sin(x) + \beta cos(2x); \alpha, \beta \in \mathbb{R}\}$. $v(x) = sin(2x) - 3cos(x)$ and $V = L^2(-\pi, \pi)$.
   The solutions are actually quite simple so spend some time on establishing the proper background for this problem. For example, do we know that K is indeed s subset of V; is it a subspace (affirmative for both cases), what is its orthonormal basis. Is $v \in V$ etc.

6. Solve 3-3.1