#15: <u>Def</u> $c = \max S$ iff $c \in S$ and c is an upper bound for S.

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HW 1

<u>Lemma</u> S has a maximum iff it is bounded above and $\sup S \in S$.

<u>Proof</u> "$\Rightarrow$" Let S have a maximum c. Then from def. c is an upper bound $\Rightarrow$ S is bounded from above.
To show $\sup S \in S$ we show $c = \sup S$. We must have $x \le c$, $\forall x \in S$, hence $\sup S \le c$ ($\sup S$ is the l.u.b.).
Since $c \in S$, we have $c \le \sup S$ ($\sup S$ is an upper bound).
From $\sup S \le c$ and $c \le \sup S$ we conclude $c = \sup S$.

"$\Leftarrow$" Let S be bounded from above, and $\sup S \in S$.
Set $c = \sup S \in S$. This c satisfies the <u>Def</u> above hence $c = \max S$, and S has a maximum.

q.e.d.