

Multiphysics Coupling of Three-Phase and Two-Phase Models of Flow in Porous Media

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Abstract. In this paper we show that multiphysics couplings for the simulation of multiphase flow can be realized without loss of accuracy and with decreased computational complexity when compared to single model codes. Specifically, we address the couplings of oil-gas-water models of flow in porous media. An interface is introduced between three-phase and two-phase regions and the problem is solved by combining local simpler solutions coupled with conditions across the interface. Here we discuss issues of matching of variables across the interface and show computational results which demonstrate efficiency and accuracy of the coupling.

1 Introduction

In geosciences, various applications have relied on multiphysics couplings which connect models or algorithms in adjacent or in overlapping computational domains. As examples of the latter, multiphase flow models and the geomechanics, or reactive transport models have been coupled together [5,9].

In this paper we address couplings of oil-gas-water models of flow in adjacent domains in porous media. Our method falls into the category of *heterogeneous domain decomposition* methods, which include, e.g., *fluid-structure interactions*. In contrast to the latter, in our algorithm, the interface across which the coupling of models occurs, is not a physical interface but rather it is one introduced for computational reasons. Specifically, it is fairly common that in a reservoir $\Omega \subset \mathbb{R}^3$ some phases or some components are absent in large parts of Ω over large time intervals $I = (0, T)$. For example, due to gravitational separation, free gas (*gas phase*) typically collects at the top of reservoir. Traditional algorithms simulate the flow by applying a model that can handle a superset of all phases and components. In our multiphysics approach we apply locally simpler models wherever appropriate. For the global solution, coupling conditions are defined. This approach allows the reduction of computational time.

In what follows, we consider an isothermal, three-phase *black-oil* model, and a simple two-phase *oil-water* model [6]. Their coupling was first defined and discussed in [8,3]. Similar issues for the coupling of the oil-water and single-phase (water) models were discussed in [7]. For details on discretization using expanded mixed finite element methods and mortar spaces, fully implicit solver issues, as well as multiscale features see [1,10] and refs. therein.

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Below we briefly formulate the models, discuss the coupling, and present computational results which were obtained using the multiphase flow and transport framework IPARS, see [11] and refs. therein. We would like to acknowledge Qin Lu, Mary F. Wheeler and John A. Wheeler for discussions related to this work.

2 Multiphase flow models

Here we consider three components $M = W, O, G$: water, oil (or an ensemble of heavy hydrocarbons), and gas (or an ensemble of light hydrocarbons) [4], respectively, in reservoir of porosity $\phi(x)$ and permeability $K(x)$, with gravity g , and depth $D(x)$, $x \in \Omega$. We use the mass conservation equation

$$\frac{\partial}{\partial t}(\phi N_M) + \nabla \cdot \mathbf{U}_M = q_M, \quad x \in \Omega, \quad (1)$$

where N_M denotes the component's concentration, and \mathbf{U}_M, q_M denote its flux and source(s), respectively. Equation (1) is a special case of general multicomponent / multiphase equations given in [6,2].

These three components can be present in one or more of the three fluid *phases* denoted by subscript $m = w, o, g$ (water, oleic, gaseous). The distribution between phases is described using η_{Mm} which denotes the mass fraction of component M in phase m . Phase saturations, relative permeabilities, pressures, viscosities, densities, and reference densities, are denoted by $S_m, k_m, P_m, \mu_m, \rho_m, \rho_{mr}$, respectively. We say that a phase m is absent if $S_m = 0$, and we refer to [7] for discussion of residual conditions and phases that are present but immobile.

We use the multiphase extension of Darcy's law for the momentum equation of phase m which defines the volumetric phase velocity \mathbf{V}_m :

$$\mathbf{V}_m = -\frac{Kk_m}{\mu_m} (\nabla P_m - \rho_m g \nabla D). \quad (2)$$

The saturations must satisfy the constraint: $S_g + S_o + S_w = 1$, and capillary pressure data is given for the differences $P_o - P_w$ and $P_g - P_o$. The model is complete when concentrations, fluxes and densities are defined. In most cases, $N_M = \sum_m \eta_{mM} S_m \rho_m / \rho_{*r}$; $\mathbf{U}_M = \sum_m \eta_{mM} \mathbf{V}_m \rho_m / \rho_{*r}$ where $*$ is a specially chosen phase. However, definitions, units, and constraints on the distribution of components between phases, vary between models. Here we are interested in a black-oil, two-phase and single-phase models.

Black-oil model. Here we assume that $\eta_{Ww} = 1, \eta_{Gg} = 1$ and that $\eta_{Oo} + \eta_{Go} = 1, \eta_{Oo} > 0$. We use formation volume factors $B_m(P_m) = \rho_{mr} / \rho_m$ for $m = w, g$. We also define $B_o = \rho_{or} / \eta_{Oo} \rho_o$. The amount of G in o at a given pressure is represented using the *gas/oil ratio* $R_o = \eta_{Go} \rho_{or} / \eta_{Oo} \rho_{gr}$ which is less than or equal to the *saturated gas/oil ratio* $R_{so}(P_o)$ given as reservoir data. We consider a *phase change* occurring in component G at *bubble pressure* P_{sat} , at which $R_o = R_{so}(P_{sat})$. At $P_o < P_{sat}$, part of the gas

component enters gas phase, or vice-versa, if $P_o > P_{sat}$, all gas component dissolves in oil phase. The concentrations and mass fluxes are defined as $N_M = S_m/B_m$, $\mathbf{U}_M = \mathbf{V}_m/B_m$, for those components $M = W, O$, which are naturally associated with their phases $m = w, o$. For gas component, we have $N_G = S_g/B_g + S_o R_o/B_o$ and $\mathbf{U}_G = \mathbf{V}_o R_o/B_o + \mathbf{V}_g/B_g$.

To complete the formulation, we define the oil phase density which depends on the pressure and on the amount of dissolved gas. Usually, reservoir data is available [2,4] for the formation volume factor of oil i) without any gas dissolved B_{do} and ii) in saturated conditions B_{so} . If $P > P_{sat}$, then $\rho_o = (\rho_{or} + R_o \rho_{gr})/B_o$, where the compressibility of the oil-gas mixture is given as $B_o = B_{do} + (B_{so}(P_{sat}) - B_{do}(P_{sat}))R_o/R_{so}$ and we have here $R_o = N_G/N_O$. If $P_o < P_{sat}$, then $S_g > 0$, $R_o = R_{so}$ and $\rho_o = (\rho_{or} + R_{so} \rho_{gr})/B_{so}$.

Two-phase model. The popular, immiscible, two-phase oil-water model can be seen as a subset of the black-oil model with no gas component present ($\eta_{Go} = 0, \eta_{Oo} = 1$) where obviously $S_g = 0, k_g = 0$. Oil and water are slightly compressible: $\rho_m = \rho_{mr} \exp(c_m P_m)$ with constants c_m . Concentrations are $N_M = S_m \rho_m / \rho_{mr}$ for $m = o, w$ and $M = O, W$, respectively.

Single-phase model. Here additionally we set $S_o = 1$.

3 Interface coupling of oil-water model with black-oil model

Consider $t \in I$ and a decomposition of $\Omega = \tilde{\Omega}^2(t) \cup \tilde{\Omega}^3(t)$ defined as follows: $\tilde{\Omega}^2(t) = \{x : S_g(x; t) = 0\}$, $\tilde{\Omega}^3(t) = \Omega \setminus \tilde{\Omega}^2(t)$. Note that at a given t , each of $\tilde{\Omega}^2(t)$, $\tilde{\Omega}^3(t)$ (but not both) may be empty if the pressure is low (high) enough. Define the *free boundary* $\tilde{\Gamma}_{23}(t) = \partial \tilde{\Omega}^2(t) \cap \partial \tilde{\Omega}^3(t)$ between the two- and three-phase region. In general, it is difficult to trace $\tilde{\Gamma}_{23}(t)$ in numerical computations.

Let Ω_h be a union of the finite elements covering Ω . For simplicity assume $\Omega_h = \Omega$. Now consider the time-independent decomposition of $\Omega_h = \Omega_h^2 \cup \Omega_h^3$ where $\Omega_h^2 \subseteq \bigcap_{t \in (0, T)} \tilde{\Omega}^2(t)$. The actual definition of Ω_h^2 is allowed to vary, see below. For completeness, $\Omega_h^3 = \Omega_h \setminus \Omega_h^2$. Define the interface $\Gamma_h^{23} = \partial \Omega_h^2 \cap \partial \Omega_h^3$.

The whole idea of multiphysics coupling between the two-phase model and the black-oil model is in applying each of these models, respectively, in Ω_h^2 and Ω_h^3 , coupled by interface conditions on Γ_h^{23} . Note that Ω_h^2 can be chosen in any way that is convenient for modeling or computational purposes. In particular, it is desirable for Ω_h^2 to be as large as possible in order to save computational time. Furthermore, we choose Ω_h^2 so that Γ_h^{23} has a convenient geometry. In particular, when using mortar spaces [1] to resolve non-matching grids over Ω_h^2 and Ω_h^3 , it is advantageous for Γ_h^{23} to have the most ‘‘straight’’ pieces possible to allow for multiscale and adaptive approximations [10].

In analogy to the above, we can define $\tilde{\Omega}^1(t) =: \{x : S_w(x; t) = 1\}$ and $\Omega_h^1 \subseteq \bigcap_{t \in (0, T)} \tilde{\Omega}^1(t)$. These single-phase regions are used in computational examples shown below. Note that $\tilde{\Omega}^2(t) \supseteq \tilde{\Omega}^1(t)$, $\Omega_h^2 \supseteq \Omega_h^1$.

Coupling conditions. Here we use $\Gamma =: \Gamma_h^{23}$. Note that Γ is “artificial”, i.e., it does not follow any material boundaries. Therefore, we apply the same conservation principles of momentum and mass on $\Gamma =: \Gamma_h^{23}$, as those applied across boundaries of finite elements. These are realized, respectively, by matching of phase pressures $P_m^2|_\Gamma = P_m^3|_\Gamma, \forall m$, and by matching of component mass fluxes. In consequence, from capillary pressure relationships, phase saturations must match (unless rock types do not match, see [3]), and we have to account properly for the existence of phases and components on each side of Γ . In particular, phase densities computed on Γ from the side Ω_h^2 must match those from the side Ω_h^3 .

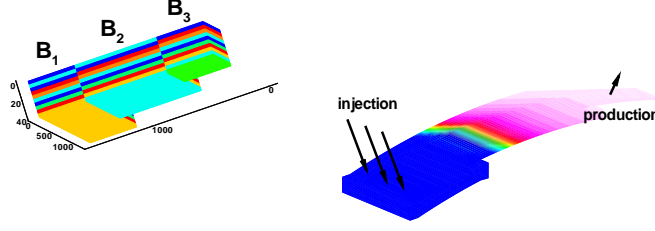


Fig. 1. Left: permeability field and geometry. Right: oil concentration contours at $t = 0$ and well locations indicate that $\Omega^2(t) \supset B_1 \cup B_2$. Gas saturation (not shown) indicates that $\Omega^3(t) \subset B_3$

Consider $x \in \Gamma$ and the amount of gas at x . Obviously, $P_o(x) < P_{sat}(x)$. During simulation, we assume that $R_o(x, t)|_\Gamma \approx R_o(t)$. This is reasonable if the reservoir is not too thick: for a typical reservoir (100' thick), the difference is around 1% and for a thick reservoir (300' thick), it can reach 2%. Furthermore, we assume that this ratio remains approximately constant in time, i.e., $R_o(t)|_\Gamma \approx R_o^\Gamma$ with R_o^Γ known at initialization. Finally, we assume that the oil phase in Ω_h^2 contains gas component with a fixed gas/oil ratio $\approx R_o^\Gamma$. Consequently, we need to find ϱ_{or}^2 and c_o^2 on Ω_h^2 such that over a certain range of pressures (P_{min}, P_{max}) , $x \in \Gamma$,

$$\varrho_{or}^2 \exp(P_o c_o^2) = \varrho_o^2|_{\Omega_h^2} \approx \varrho_o^3|_{\Omega_h^3} = \frac{(\varrho_{or}^3 + R_o^\Gamma \varrho_{gr}^3)}{B_o(P_o)}, P_o \in (P_{min}, P_{max}). \quad (3)$$

Given $R_o^\Gamma, B_o, \varrho_{mr}^3$, the parameters ϱ_{or}, c_o can be found, for example, by a least squares procedure. Obviously, for $R_o^\Gamma > 0$, the quality of such a match depends, in particular, on the size of (P_{min}, P_{max}) . From (3) it follows that the concentrations (and fluxes) computed on each side of the interface match only if a conversion factor is applied. To see that, multiply (3) on Γ by the saturations $S_o^2|_\Gamma = S_o^3|_\Gamma$ to get

$$\hat{\varrho}_{or}^2 N_o^2 = S_o^2 \hat{\varrho}_{or}^2 \exp(P_o \hat{c}_o) \approx S_o^3 \frac{(\varrho_{or}^3 + R_o^\Gamma \varrho_{gr}^3)}{B_o(P_o)} = (\varrho_{or}^3 + R_o^\Gamma \varrho_{gr}^3) N_o^3.$$

Similar matching is obtained for fluxes, using the fact that in two-phase conditions, $k_o^2(S_o^2)|_\Gamma = k_o^3(S_o^3)|_\Gamma$.

4 Examples

Here we present simulation results illustrating the concepts discussed above. The computational domain $\Omega_h = B_1 \cup B_2 \cup B_3$ is presented in Fig. 1. We use the mortar approach to resolve nonmatching grids and an iterative solver on Γ . Γ is a subset of the union of faces between B_1, B_2, B_3 . Simulations can be performed either using a traditional approach $\Omega_h^3 \equiv \Omega_h$ denoted by **b**, or using the domain decomposition approach in several variants. The most natural choice (and one which proves to be most efficient) is one in which we assign $\Omega_h^1 \equiv B_1$, $\Omega_h^2 \equiv B_2$ and $\Omega_h^3 \equiv B_3$ (we shall call it **shb**).

Now we discuss results in Fig. 2. First, we compare the solutions obtained **b** with those obtained with **shb**. It turns out that their agreement is excellent, regardless of how much gas is present in the reservoir. Next, we consider

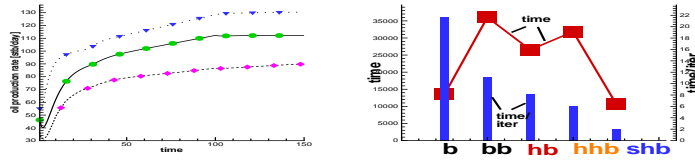


Fig. 2. Left: comparison of the oil production rates obtained with multiphysics (symbols only) and with single model (dotted, solid and dashed lines, respectively). From top to bottom: in block B1: unsaturated oil; saturated oil but no gas cap; gas cap. Right: efficiency of multiphysics couplings

the computational efficiency of the multiphysics procedure. In addition to scenarios discussed above, we consider scenario **hb** in which $\Omega_h^2 \equiv B_1 \cup B_2$ with $\Omega_h^3 \equiv B_3$, as well as two additional scenarios: i) **bb** which is like **hb** with black-oil model functionally replacing two-phase model, and ii) **hnb** which is like **shb** in which two-phase model functionally replaces the single-phase model over B_1 .

As expected, the time per iteration which corresponds to total subdomain time decreases, as more simple models are used. However, the number of interface iterations increases with the number of subdomains. Therefore, the total time does not decrease monotonically, but multiphysics **shb** is faster than traditional **b** simulation. In general, the efficiency of multiphysics depends mainly on the size of subdomains associated with individual models and on the number of iterations necessary to achieve a desired accuracy of the coupling.

Finally, we study sensitivity to the matching of densities: Fig. 3 contains plots of $\varrho_o^3|_\Gamma$ and of $\varrho_o^2|_\Gamma$ for two different intervals (P_{min}, P_{max}) used in

the matching procedure. When this interval is small, the multiphysics results match very well those obtained by traditional simulation. When the interval is large, a large discrepancy due to the density modeling error may occur.

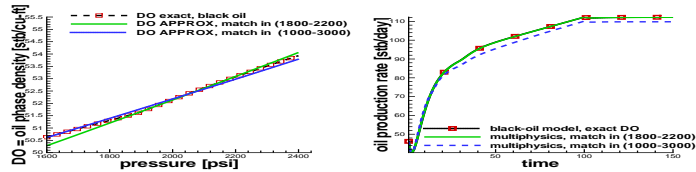


Fig. 3. Sensitivity to density matching. Left: density of oil (exact and approximated using (3)). Right: corresponding well rates

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