HOMEWORK PROBLEMS

Basic Skills

Q10B.1 Suppose we confine an electron to a box whose length is roughly equal to an atom's diameter ($\approx$0.2 nm). If we perform an experiment to evaluate the electron's energy, what are the three lowest values we could get?

Q10B.2 Suppose we confine an electron to a very thin wire 5.0 nm long. We do an experiment to evaluate the electron's electron and find it to be 0.376 eV. What is the value of n for the electron's energy eigenfunction after the experiment is over?

Q10B.3 Suppose we can model an atomic nucleus as a box for protons and neutrons whose length is equal to the nucleus' diameter ($\approx$8 fm $= 8 \times 10^{-15}$ m). If we do an experiment to measure the energy of a neutron in such a nucleus, roughly what is the smallest possible result? (*Hint:* mc^2 for a neutron is about 939 MeV.)

Q10B.4 Verify that the Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/me^2$ is indeed about 0.053 nm.

Q10B.5 A cyanine dye molecule has a spine of carbon atoms along which electrons can freely travel, but which has barriers at each end of the spine that an electron cannot get past. We can model an electron in this molecule as if it were a quanton in a box. If the distance between barriers is 0.93 nm, what are the lowest three energies that a spine electron might have?

Q10B.6 Imagine that the wavefunction for an electron in a hydrogen atom is equal to the $n = 7$ energy eigenfunction for that atom. What is the electron's energy? What is the approximate radius of its effective "orbit"?

Q10B.7 One way in which a water molecule can vibrate has a measured angular frequency of $\omega = 3.0 \times 10^{14}\ \text{s}^{-1}$. What is the value of $\hbar\omega$ in eV for this system if we model it as a simple harmonic oscillator? What are its lowest three possible energy values?

Q10B.8 The separation between the atoms in an HCl (hydrochloric acid) molecule can vibrate with a measured angular frequency of $\omega = 5.6 \times 10^{14}\ \text{s}^{-1}$. What is the value of $\hbar\omega$ in eV for this system if we model it as a simple harmonic oscillator? What are its lowest three possible energy values?

Q10B.9 Write down the harmonic oscillator energy eigenfunction for $n = 6$. (You may leave factors of b and c_0 in your final result.)

Q10B.10 Write down the harmonic oscillator energy eigenfunction for $n = 7$. (You may leave factors of b and c_1 in your final result.)

Modeling

Q10M.1 Consider a photon in a perfectly mirrored box of length L. This is a quanton-in-a-box problem, but the results in section Q10.3 do not apply because our derivation there assumed that the quanton was nonrelativistic. The main difference between a photon and a nonrelativistic quanton is the relationship between energy and momentum: for a photon, $E = |\vec{p}|c$ (in SI units), but for a nonrelativistic quanton, $E = |\vec{p}|^2/2m$. Use this to determine the possible energies of a photon in a perfectly mirrored box.

Q10M.2 Consider a nitrogen (N_2) molecule bouncing around a box 10 cm on a side. Pretend that the molecule can only move in one dimension. Note that the mass of an N_2 molecule is roughly 28 times that of a proton.

(a) What is the approximate value of n for the molecule's energy eigenfunction if it has an energy $E_n \approx 0.025$ eV that one would expect from random thermal motion?

(b) Estimate the ratio $\Delta E/E_n$ for this value of n, where ΔE is the energy difference between adjacent possible energies in the neighborhood of E_n.

(c) Would the energies available to this molecule *seem* continuous, as classical mechanics would predict?

Q10M.3 Imagine that the proton and electron in a hydrogen atom were not charged. They could still form an atom based on the gravitational (instead of electrostatic) attraction between the two quantons. What would be **(a)** the minimum orbital radius and **(b)** the minimum energy of the electron in this gravitational atom?

Q10M.4 The potential energy of the strong interaction between two quarks has the approximate form $V(r) = br$, where b is some constant. Assume we have a light quark of mass m interacting with a very massive quark. Using the same basic approach we used for the Bohr model, find the possible energies and orbital radii of this system. If the light quark's rest energy $mc^2 = 310$ MeV for the light quark and $b \approx 15$ tons $\approx 150{,}000$ N (these approximate the actual values for this system), estimate the minimum radius of the lighter quark's "orbit." (*Hints:* The magnitude of the force exerted on the little quark in its orbit is given by $F = |dV/dr| = b$. You should find that $E_n \propto n^{2/3}$.)

Q10M.5 Suppose the potential energy associated with the interaction between an electron and some fixed system is a three-dimensional version of the harmonic oscillator potential energy function: $V(r) = \frac{1}{2}m\omega^2r^2$, where r is the distance between the movable quanton and the origin.

(a) Using the same assumptions and approach we took in the Bohr model for this system, find an expression for the possible energies of this system.

(b) How does your result compare to that for the one-dimensional harmonic oscillator (see equation Q10.19)?

(*Hint:* The magnitude of the spring-like force on the movable quanton is $|\vec{F}| = |dV/dr| = m\omega^2 r = k_s r$.)