# Homework 6

Applied quantum mechanics

(updated Thursday Feb 28, clarifications for 3, 4 and 6)

Due Friday February 23 at 5pm

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1. The Figure below shows a simplified model of the electrostatic potential energy for *some* of the electrons in a cyanine dye molecule. The chemical structure of the dye molecule is shown directly underneath the graph of $V(x)$. The bond length between atoms is 0.16 nm. There is one electron from each central carbon atom that "experiences" this $V(x)$ landscape. These electrons are essentially free to move in the x-direction, but cannot move beyond the nitrogen atoms. Underneath the $V(x)$ graph, I've drawn some of the wavefunctions that satisfy the Schrodinger Equation for electrons in this $V(x)$.
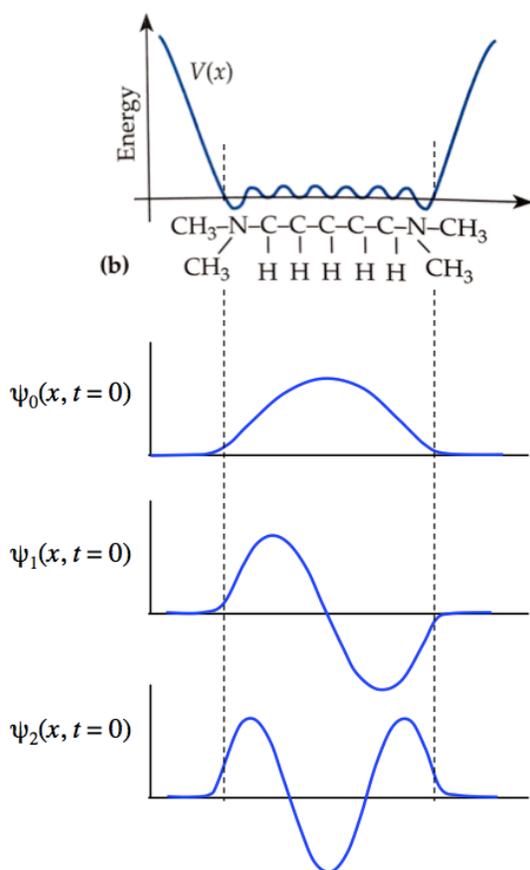
a) Estimate the energy of an electron when it has wavefunction ψ_0. You can get the energy correct to within ±10% by measuring the deBroglie wavelength and calculating kinetic energy.

b) Do the same for wavefunction ψ_1

c) Do the same ψ_2.

d) Draw an energy level diagram for these first three wavefunctions. Label the energies in units of electron-Volts.

2. Quantum Harmonic Oscillator
a) Q10B.7 from Unit Q, 3rd edition.

b) An **optional** question about nanotechnology research:
   Q10M.8 from Unit Q, 3rd edition.

3. Vibrational energy in a diatomic gas
A nitrogen nucleus in an N_2 molecule behaves like a harmonic oscillator. When N_2 is vibrating, the two nuclei move symmetrical towards/away from each other along the axis of the chemical bond. The analysis is simplest if we imagine one nitrogen atom connected via a spring to the plane bisecting the chemical bond. This analogous "mass-on-a-spring" system has a spring constant $\kappa = 6000$ N/m.

Note: If you are unfamiliar with the mass on a spring system, please read about it here:
        https://en.wikipedia.org/wiki/Harmonic_oscillator#Simple_harmonic_oscillator

a) What is the smallest amount of energy that can make this quantum harmonic oscillator transition to a higher energy level? Give your answer in units of electron volts.

b) What temperature should we make the N_2 gas so that the average translational kinetic energy of each molecule, $(3/2)k_B T$, is equal to the energy difference between quantum harmonic oscillator energy levels?

4. Photons emitted from a quantum systems
   a) Q11B.4 from Unit Q, 3rd edition.
   b) Q11B.5 from Unit Q, 3rd edition.
   c) Q11.B6 from Unit Q, 3rd edition.

   Note: When light of a specific wavelength is emitted, the light is called an "emission line". This terminology arose amongst scientists using spectrometers to study light. Monochromatic light looks like a sharp line in a spectrometer.

5. Fermi estimate - Energy requirements for lighting.

David McKay's book suggests that lighting is a very small fraction of the 200 kWh/d energy budget for the average U.S. citizen. Let's check his reasoning by considering streetlights in Corvallis (population ~ 60,000).

Make a simple model of Corvallis. Treat the streets as a regular grid. Streetlights are spaced out evenly. Make an educated guess regarding the geometry of the grid and the spacing between streetlights. Each streetlight uses electrical energy at a rate 300 J/s.

In Corvallis, how much energy is used per resident per day by these street lights?
Sense making: Compare to overall energy budget of 200 kWh/d per person.

6. Infrared light interacting with gases in the atmosphere

This question is based on the climate modeling software called MODTRAN published on the internet by David Archer. MODTRAN calculates how much infrared (IR) radiation leaves the Earth's atmosphere and travels away into outer space. This quantity is called "upward IR heat flux".

http://climatemodels.uchicago.edu/modtran/
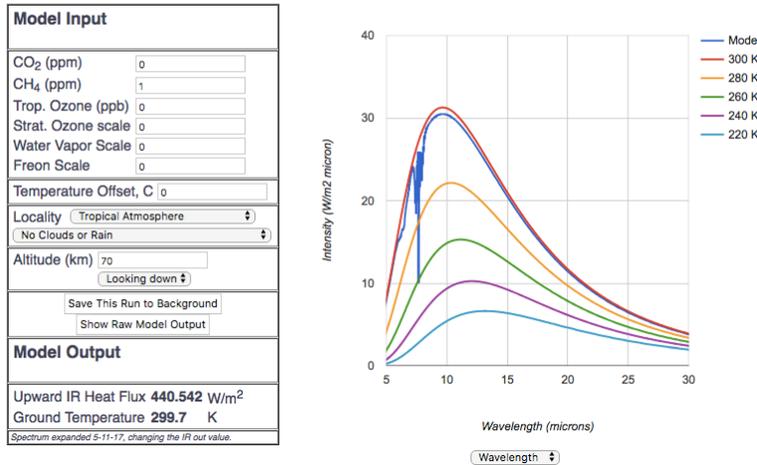
For additional information about greenhouse gases, see Chpt 4 of Archer's textbook:
http://forecast.uchicago.edu/chapter4.pdf

Methane has a current concentration of 1.7 parts per million (ppm) in the atmosphere and it is doubling at a faster rate than CO_2.

a) Would an additional 10 ppm of methane in the atmosphere have a larger or smaller impact on the "upward IR heat flux" than an additional 10 ppm of CO_2 at current concentrations? (*Use the computer model to find out*)

b) Where in the spectrum does methane absorb? What concentration does it take to begin to saturate the absorption in this band? Explain what you are looking at to judge when the gas is saturated.

c) Would a doubling of methane have as great an impact on the upward IR heat flux as a doubling of CO_2?

d) What is the "equivalent CO_2" for a small change in methane concentration? For example, if I increase methane by 1 ppm, how many ppm of CO_2 would lead to the same change in upward IR heat flux?