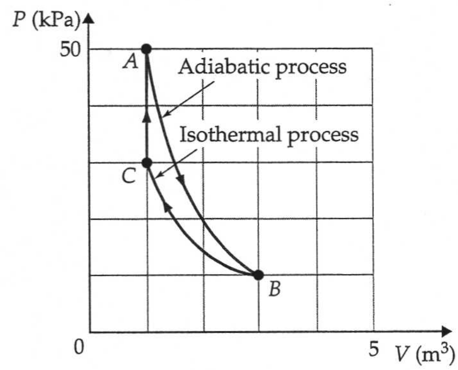
T7M.5 Suppose we constrain a gas to follow the three-step cyclic process shown in the graph below. Prepare a chart (like the one shown in example T7.1) that specifies the sign of Q, W, and ΔU for each step in the process.



T7M.6 Heat must flow into the gas during the process $C \rightarrow A$ shown in the drawing associated with problem T7M.5. Why? If the gas's temperature is 290 K at point C, find its temperature at point A and the heat that has flowed into the gas in this process if the gas is monatomic.

T7M.7 A bubble of air forms at the bottom of the ocean floor 66 ft below the surface, where the ambient pressure is about 300 kPa = 3 atm. The bubble has an initial volume of about 25 cm^3 and a temperature of 8°C. If the bubble rises to the surface so fast that it expands essentially adiabatically, what is its final volume and temperature?

T7M.8 A research balloon bound for the stratosphere is filled at sea level with 800 m^3 of helium whose initial temperature is 285 K. The balloon is released, and it climbs to an altitude where the air pressure is 0.045 times its value at sea level.

(a) If the helium expands adiabatically, what is the balloon's volume at its final altitude?

(b) What is the helium's final temperature?

T7M.9 An adiabatic expansion process begins with 0.1 mole of gas at a pressure of 100 kPa and ends with a pressure of 40 kPa, while its temperature falls from 300 K to 208 K.

(a) Is the gas diatomic or monatomic?

(b) How much work did the gas do in this process?

T7M.10 Suppose that while pumping up a bike tire, we fairly rapidly compress 1500 cm^3 of air from atmospheric pressure and room temperature to a pressure of about 5 atm (which is about 60 psi above atmospheric pressure, which is what a tire gauge would read).

(a) What is this packet of air's volume as it enters the tire?

(b) What is its final temperature?

(c) How much work did we do to compress it?

T7M.11 We measure a certain gas's *specific heat* (not its heat capacity) at constant volume to be 730 J/(kg·K) and its specific heat at constant pressure to be 1020 J/(kg·K). Is this gas monatomic or diatomic?

T7M.12 Water has a specific heat of 4186 J/kg.

(a) Calculate the effective number of "degrees of freedom" f for water (see equation T7.17).

(b) Speculate physically about why this number might be as large as it is.

Derivation

T7D.1 Use the ideal gas law to show that if $PV^\gamma =$ constant for an adiabatic process, then $TV^{\gamma-1} =$ constant for such a process.

T7D.2 Use $PV^\gamma = P_i V_i^\gamma$ to show that the work done during an adiabatic volume change from V_i to V_f is

$$W = \frac{P_i V_i}{\gamma - 1}\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1} - 1\right] \tag{T7.25}$$

(*Hint:* Solve $PV^\gamma = P_i V_i^\gamma$ for P as a function of V, and then use equation T7.6.)

T7D.3 One can find the work involved in an adiabatic process in one of two ways. The first way is to use equation T7.25. The second way is to realize that since no heat flows into the gas, the work that flows into (or out of) the gas is the same as the gas's change in internal energy: $W = \Delta U = Nk_B \Delta T$. Prove mathematically that this second method yields the same result as equation T7.25.

T7D.4 How does T vary with P in an adiabatic process?

T7D.5 One can also derive the adiabatic gas law from a molecular-level perspective. This problem outlines how. Suppose a piston with area A confines a gas to a cylinder of length L, and suppose the piston moves slowly inward with a speed $|\vec{u}| = -dL/dt$ (negative because L is decreasing). Let's define our coordinate system so that the piston moves in the $-x$ direction.

(a) If the piston were at rest, a molecule hitting it would simply reverse its x-velocity. But if the piston is moving inward, argue that the absolute value of the molecule's x-velocity *increases* by $2|\vec{u}|$. (*Hint:* One can do this either by transforming to the frame in which the piston is at rest and then back again, or by solving the one-dimensional elastic collision problem for a light object hitting a much more massive moving object.)

(b) Show that this means that the kinetic energy of a molecule with x-velocity v_x increases by $dK \approx 2m|\vec{u}||v_x|$ per collision in the limit that $|\vec{u}| \ll |v_x|$.

(c) Assuming that the molecule does not interact with other molecules, the time between collisions with the piston will be $2L/|v_x|$. Show then that the rate at which the molecule's energy will increase with time is

$$\frac{dK}{dt} = \frac{m|\vec{u}||v_x|^2}{L} \tag{T7.26}$$

(d) The total amount of energy that the gas gains from the piston is therefore the number of molecules N times the value of the right side of equation T7.26 averaged over all molecules. Argue that this will be