Homework #7

Due on March 3 at 5pm. Hand in to Paul Emigh (office 491 Weniger).

1. Spectral intensity of black body radiation

Consider the equation for the spectral intensity emitted by a blackbody source.

$$I_\lambda(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

a) Differentiate the spectral intensity equation to find the wavelength associated with maximum spectral intensity. Give your answer in terms of $k_B T$ and a numerical constant.

> *Hint*: To streamline the algebraic manipulation, try writing the function as
>
> $$A\lambda^{-5}(e^{B/\lambda} - 1)^{-1},$$
>
> where A and B represent constants. Then apply the product rule and chain rule.

b) Integrate the spectral intensity equation to show that total intensity, $I = \sigma T^4$, and use the dimensionless definite integral

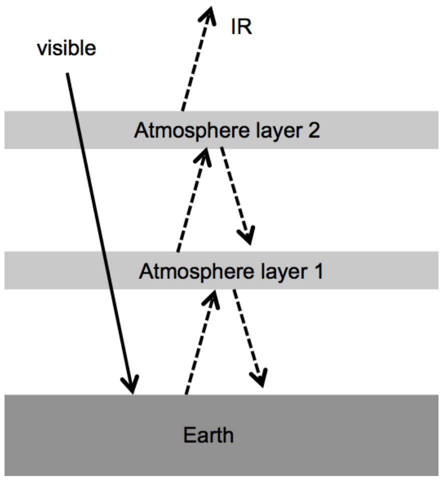
$$\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{\pi^4}{15}$$

to relate σ to fundamental constants.

> *Hint*: This question is giving you practice with "integration by substitution", also known as "u-substitution".

c) The universe is filled with thermal radiation which has a blackbody spectrum at an effective temperature of 2.7 K (see Chapter 15 of "Modern Physics" by Krane). What is the peak wavelength of the radiation? In what region of the electromagnetic spectrum is this peak wavelength?

2. A two-layer model for estimating the Earth's temperature



The Figure shows a two-layer model, similar to the one-layer model we did in class. Both atmospheric layers are transparent to visible light, but a blackbody for IR.

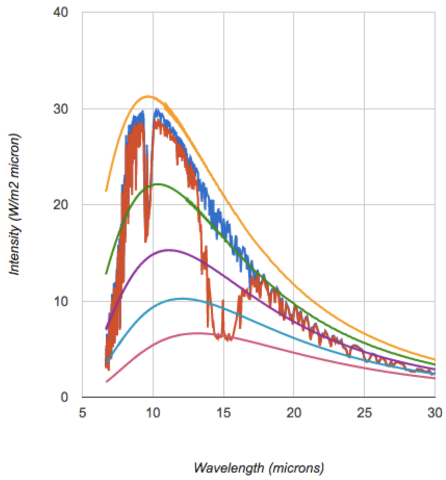
a) Write the energy budgets for both atmospheric layers, for the ground, and for the earth as a whole, just as we did for the one-layer model.

b) Manipulate the budget for the Earth as a whole to obtain the temperature T_2 of atmosphere layer 2 (the upper layer). This temperature is known as the skin temperature.

c) Insert the value you found for T_2 into the energy budget for layer 2, and solve for the temperature of layer 1 in terms of T_2. How much bigger is T_1 than T_2?

d) Now insert the value you found for T_1 into the budget for atmospheric layer 1, to obtain the temperature of the ground, T_{ground}. Is the greenhouse effect stronger or weaker because of the second layer?

3. Changing CO_2 in the atmosphere



http://climatemodels.uchicago.edu/modtran/

The University of Chicago simulation uses the optical absorption cross-section of CO_2 and other atmospheric gases to calculate the upward flux of energy carried by electromagnetic radiation away from the earth. The model also takes into account the temperature profile of the atmosphere. The graph shows the intensity spectrum you would observe from a satellite pointing towards the dark (night-time) side of the earth.

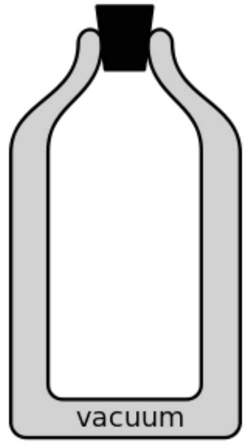
a) Use the simulation to calculate the total upward flux (measured in W/m^2) for various CO_2 concentrations. Keep all other variables fixed. I suggest setting the CO_2 to 0, 3, 10, 30, 100, 300, and 900 ppm. Record the total upward flux, and notice how the spectrum changes as CO_2 is increased.

Note: The current concentration of CO_2 in the atmosphere is 406 ppm. Before the industrial revolution it varied between 200 and 280 ppm (the ice age cycles).

b) Make a plot of total upward flux vs. CO_2 concentration and discuss the shape of curve. Why is total upward flux very sensitive to increasing CO_2 when CO_2 concentration is low, but less sensitive to increasing CO_2 when CO_2 concentration is high?

Note: The reading assignment from Day 14 is an excellent resource.

4. Thermos



The thermos shown above is filled with boiling hot water (T = 373 K). The thermos is 30 cm tall. The inner cylinder (which holds the water) has diameter 8 cm. The outer cylinder has diameter 12 cm. Between the inner and outer cylinders is vacuum.

The inner wall of the thermos is in thermal equilibrium with the water. The inner wall emits a blackbody spectrum carrying an outward flow of energy through the vacuum. This outward flow of electromagnetic radiation is the only way that the water can cool off.

The outer wall of the thermos is at room temperature (T = 293 K). The outer wall of the thermos emits a blackbody spectrum, which carries an inward flow of energy through the vacuum (toward the water).

a) When the hot water is still boiling hot, estimate the outward flow of energy through the vacuum.

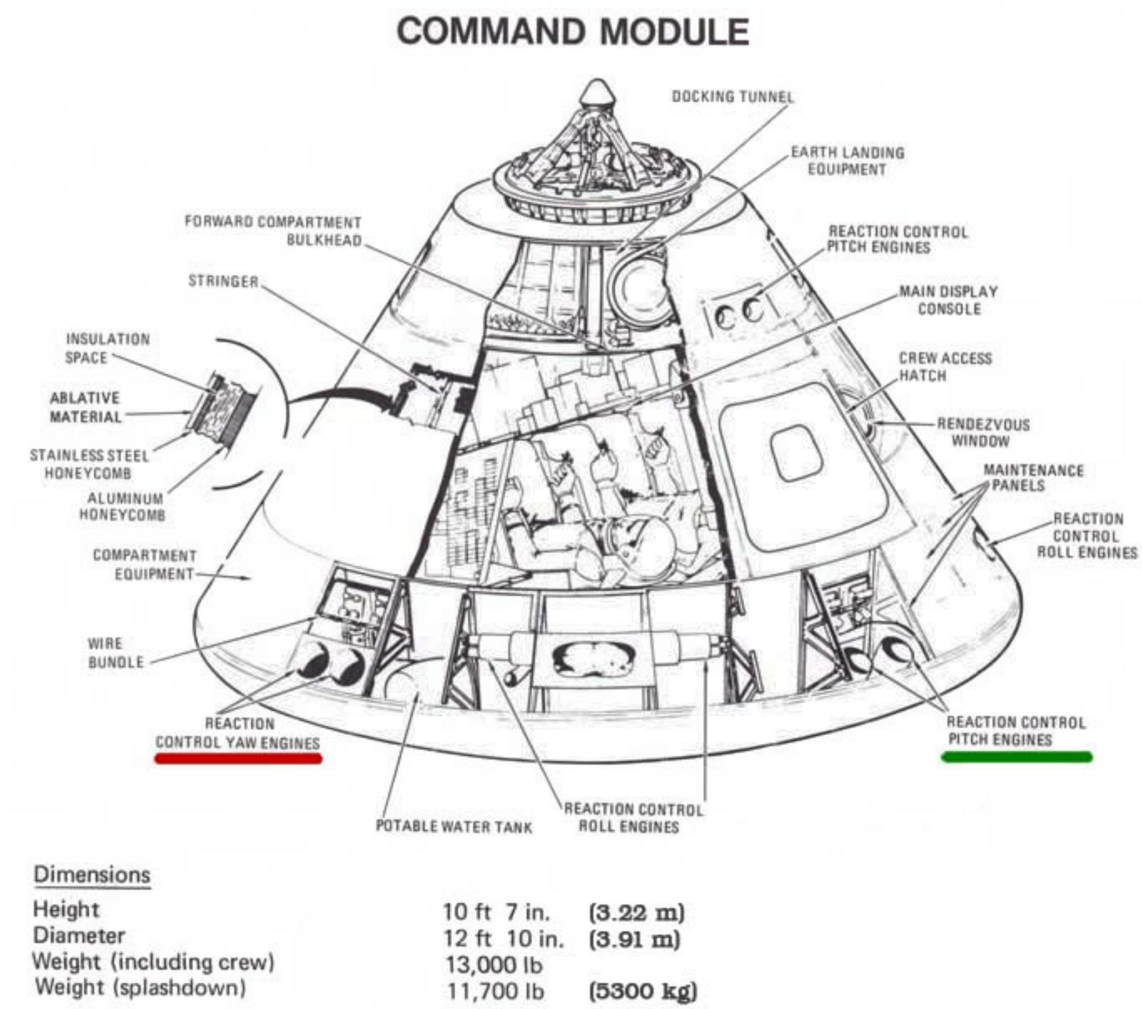
b) Calculate the inward flow of energy through the vacuum.

c) Estimate how long it will take for the water to cool to 353 K.

Note: You can do the estimate without any calculus, but if you'd like the challenge – use calculus to improve your estimate.

d) Consider a different thermos design, in which the vacuum is replaced by a high performance insulation material. The insulation material stops the electromagnetic radiation, but allows heat conduction via thermal contact. Assume the insulation material has a thermal conductivity of 0.025 W/mK. What is the rate of heat flow through this insulation material when the water is boiling hot? How does the performance of this alternative thermos design compare to the vacuum design?

5. Humans in space



a) The picture shows the Apollo command module that was used to send people to the moon. During a trip to the moon, the command module is bathed in sunlight (intensity 1350 W/m^2). Assume the module is designed to absorb 100% of the incident sunlight. The only way the command module can loose heat energy is by emitting blackbody radiation. The only other source of heat is the three astronauts who each produce 50 W. The generation of body heat is very small compared to the other heat flows in the problem and can be neglected. Calculate the equilibrium temperature of the command module.

Note: To simplify the calculation of areas, you can assume the command module is a sphere.

b) When the module flies into the shadow of the moon there is not more heat input from sunlight. The module starts to cool. Determine the rate of temperature drop (degrees Kelvin per hour) when the module first enters the shadow of the moon. Note: To answer this question you will have to estimate the relationship between the internal energy and the temperature of the module.