Day 15

Monday, February 11, 2019 11:00 AM

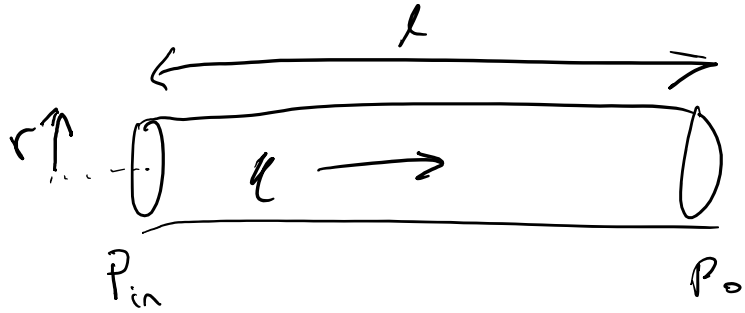
Dimensional Analysis

Ex Contemporary Challenge

1 out of 3 deaths each year in USA each year are from heart failure

Heart can't produce enough pressure to pump blood

Schematic of Blood Vessle



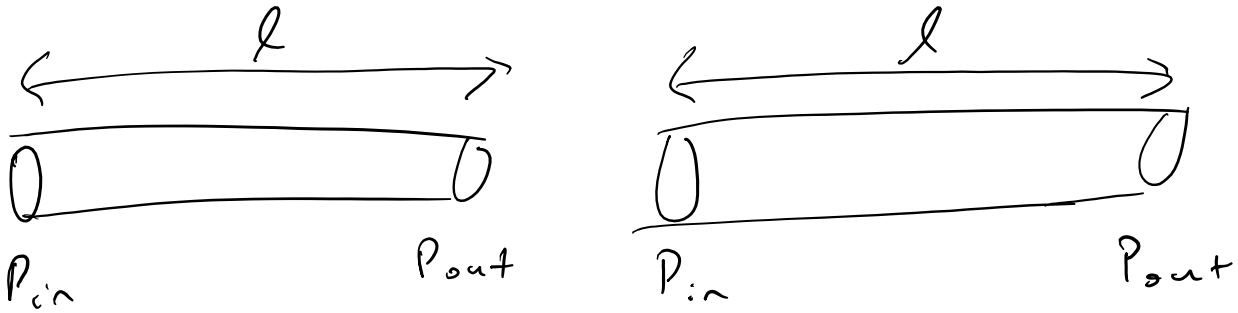
$$\Delta P = P_{out} - P_{in}$$

What's the relationship between r and ΔP

Step 1: Find dimensions of the various quantities

	symbol	unit	dimension
flow rate	Q	cm^3/s	L^3/T

flow rate	q	cm^3/s	L^3/T
Viscosity of blood	μ	$Pa \cdot s$	$\frac{M \cdot L}{T^2} \frac{1}{L^2} \cdot T$ $\hookrightarrow \frac{M}{L \cdot T}$
radius	r	cm	L
Pressure Drop	ΔP	Pa	$\frac{M}{T^2 L}$



ΔP scales with l

$$\frac{\Delta P}{l} \propto q^a \cdot r^b \cdot \mu^c$$

$$\left(\frac{M}{T^2 L^2}\right) = \left(\frac{L^{3a}}{T^a}\right) L^b \cdot \left(\frac{M^c}{T^c L^c}\right)$$

$$= \left(\frac{L^{3a}}{T^a}\right) L^b \left(\frac{M}{T L}\right)$$

$c = 1$

$a = 1$

$b = -4$

$$= \left(\frac{L}{T^a}\right)^b \left(\frac{M}{TL}\right) \qquad b = -4$$

$$\frac{\Delta P}{l} = \frac{q \cdot \mu}{r^4}$$ huge!