Example T9.2

Problem: A **heat pump** is essentially a refrigerator that moves heat from outside the house (the cold reservoir) to inside the house (the hot reservoir). Suppose that the outside temperature is 0°C and the inside is 22°C, and that the COP of the heat pump is 8.0. **(a)** Show that this COP is possible. **(b)** If the house requires 36 kW of heat energy to keep its temperature at 22°C, how much energy do we have to supply to the heat pump?

Solution **(a)** Since $T_C = 0°\text{C} = 273\text{ K}$ and $T_H = 22°\text{C} = 295\text{ K}$, the maximum possible COP for the heat pump is $T_C/(T_H - T_C) = (273\text{ K})/(22\text{ K}) = 12.4$. Therefore, a COP of 8.0 is certainly possible.

(b) The problem states that we must supply an energy $|Q_H| = 36{,}000$ J to the house every second. Since $|Q_H| - |Q_C|$ is equal to the mechanical energy W that we have to supply, and since equation T9.12 implies that $|Q_C| = (\text{COP})W$, we have

$$W = |Q_H| - |Q_C| = |Q_H| - (\text{COP})W \quad \Rightarrow \quad W(1 + \text{COP}) = |Q_H| \tag{T9.13}$$

Solving for W and substituting in the numbers yields

$$W = \frac{|Q_H|}{1 + \text{COP}} = \frac{36\text{ kJ}}{9} = 4\text{ kJ} \tag{T9.14}$$

This is the mechanical energy that we must supply each second. Therefore, if we use a heat pump, a mere 4 kJ of mechanical energy will bring 36 kJ of heat energy into the house. This is a real bargain!

If everyone would use heat pumps to heat their houses in the winter, a very large amount of energy could be saved. Unfortunately, heat pumps are quite a bit more expensive (and somewhat less reliable) than standard furnaces (and one can relatively inexpensively heat a home with natural gas, even if supplying 36 kJ of heat requires 36 kJ-worth of natural gas). This makes heat pumps less economically feasible than they should be, considering the value of conserving energy and producing less greenhouse gases.

T9.6 The Carnot Cycle

The **Carnot cycle,** which was first described by the French physicist Sadi Carnot (pronounced *car-NOH*) in 1824, is a hypothetical cyclic process that can be used to convert heat to mechanical energy by using an ideal gas as the working substance. While this particular cycle is not used in realistic engines (for a variety of reasons), it does describe an idealized sequence of gas processes that could be used in principle to produce mechanical energy at the maximum theoretical efficiency allowed by the second law of thermodynamics.

Steps in the Carnot cycle

The Carnot cycle uses an ideal gas confined in a cylinder by a piston, as shown in figure T9.5. Imagine that a flywheel keeps the piston moving in and out of the cylinder at a regular pace and stores the mechanical energy produced by the engine. The Carnot cycle consists of the following four steps:

1. Just as the flywheel begins to pull the piston out, we put the cylinder in thermal contact with a reservoir at temperature T_H, which keeps the gas temperature fixed at T_H. Since the thermal energy U of an ideal gas depends on N and T but not on V, this means that U also is fixed.