18.2.2: The perturbation Hamiltonian is

$$H' = -\mathbf{d} \bullet \mathbf{E} = -(-e\mathbf{r}) \bullet \mathcal{E}_0 \hat{\mathbf{z}} e^{-t^2/\tau^2} = ez\mathcal{E}_0 e^{-t^2/\tau^2}$$

Time dependent perturbation theory tells us that the coefficient to be in a new state $|f\rangle$ at time $t = \infty$ after starting (at time $t = -\infty$) in state $|1\rangle$ is :

$$c_f(\infty) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} \langle f|H'(t')|1\rangle e^{i(E_f - E_1)t'/\hbar}\, dt'$$

For this case, we get

$$\begin{aligned} c_{2\ell m}(\infty) &= \frac{1}{i\hbar}\int_{-\infty}^{\infty} \langle 2\ell m|H'(t')|100\rangle e^{i(E_2 - E_1)t'/\hbar}\, dt' \\ &= \frac{1}{i\hbar}\int_{-\infty}^{\infty} \langle 2\ell m|ez\mathcal{E}_0 e^{-t^2/\tau^2}|100\rangle e^{i(E_2 - E_1)t'/\hbar}\, dt' \\ &= \frac{e\mathcal{E}_0}{i\hbar}\langle 2\ell m|z|100\rangle \int_{-\infty}^{\infty} e^{-t'^2/\tau^2} e^{i\omega t'}\, dt' \end{aligned}$$

where $\omega = (E_2 - E_1)/\hbar$. The time integral can be done without knowing the final state, so let's do that first:

$$c_{2\ell m}(\infty) = \frac{e\mathcal{E}_0}{i\hbar}\langle 2\ell m|z|100\rangle \int_{-\infty}^{\infty} e^{-t'^2/\tau^2} e^{i\omega t'}\, dt' = \frac{e\mathcal{E}_0}{i\hbar}\langle 2\ell m|z|100\rangle \sqrt{\pi}\tau e^{-\omega^2\tau^2/4}$$

Now we need to calculate the spatial matrix elements $\langle 2\ell m|z|100\rangle$. Selection rules tell us that the final state must have $m = 0$ and odd parity, so only the $2p_0$ state has a non-zero matrix element (the book appears to be incorrect).

$$\langle 200|z|100\rangle = 0$$
$$\langle 21,\pm 1|z|100\rangle = 0$$

Now we need to find $\langle 210|z|100\rangle$. To calculate the matrix elements we need the following.

$$R_{10}(r) = \frac{2}{a_0^{\frac{3}{2}}} e^{-r/a_0}$$

$$R_{21}(r) = \frac{1}{2\sqrt{6}(a_0)^{\frac{3}{2}}}\frac{r}{a_0} e^{-r/2a_0}$$

$$Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta = Y_{10}^*(\theta,\phi)$$

$$z = r\cos\theta = r\sqrt{\frac{4\pi}{3}} Y_{10}(\theta,\phi) = r\sqrt{\frac{4\pi}{3}} Y_{10}^*(\theta,\phi)$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

The matrix element can be broken into radial and angular parts:

$$\begin{aligned}\langle 210|z|100\rangle &= \int_0^\infty R_{21}^*(r)\ r\ R_{10}(r)\ r^2 dr \int Y_{10}^*(\theta,\phi)\ \cos\theta\ Y_{00}(\theta,\phi) d\Omega \\ &= \int_0^\infty R_{21}^*(r)\ r\ R_{10}(r)\ r^2 dr \int Y_{10}^*(\theta,\phi)\ \sqrt{\frac{4\pi}{3}} Y_{10}(\theta,\phi)\ \frac{1}{\sqrt{4\pi}} d\Omega \\ &= \int_0^\infty R_{21}^*(r)\ r\ R_{10}(r)\ r^2 dr \frac{1}{\sqrt{3}} \int Y_{10}^*(\theta,\phi)\ Y_{10}(\theta,\phi)\ d\Omega\end{aligned}$$

The angular part is simple, leaving.

$$\begin{aligned}\langle 210|z|100\rangle &= \frac{1}{\sqrt{3}} \int_0^\infty \frac{2}{a_0^{\frac{3}{2}}} e^{-r/a_0} r \frac{1}{2\sqrt{6}(a_0)^{\frac{3}{2}}} \frac{r}{a_0} e^{-r/2a_0} r^2 dr = \frac{1}{3\sqrt{2}} \int_0^\infty \frac{r^4}{a_0^4} e^{-3r/2a_0} dr \\ &= \frac{a_0}{3\sqrt{2}} \int_0^\infty x^4 e^{-3x/2} dx = \frac{a_0}{3\sqrt{2}} \frac{4!}{\left(\frac{3}{2}\right)^5} = \frac{a_0}{\sqrt{2}} \frac{2^8}{3^5}\end{aligned}$$

Thus we get

$$c_{2\ell m}(\infty) = \frac{e\mathcal{E}_0}{i\hbar} \frac{a_0}{\sqrt{2}} \frac{2^8}{3^5} \sqrt{\pi}\tau e^{-\omega^2\tau^2/4}$$

The resultant probabilities are

$$\begin{aligned}\mathcal{P}_{200}(\infty) &= |c_{200}(\infty)|^2 = 0 \\ \mathcal{P}_{210}(\infty) &= |c_{210}(\infty)|^2 = \left(\frac{ea_0\mathcal{E}_0}{\hbar}\right)^2 \frac{2^{15}}{3^{10}} \pi\tau^2 e^{-\omega^2\tau^2/2} \\ \mathcal{P}_{21,\pm1}(\infty) &= |c_{21,\pm1}(\infty)|^2 = 0\end{aligned}$$

18.2.4: The ejected electron has energy 16 keV, which is about 1000 times larger than the energy of the bound electron. Hence the velocity of the ejected electron ($E = mv^2/2$) is about $\sqrt{1000}$ times larger than the bound electron and the time scale of the ejection (perturbation) is about $\sqrt{1000}$ times smaller than the time scale of the bound electron motion. The probability of a transition (Eqn. 18.2.13) caused by the perturbation scales like $e^{-\omega^2\tau^2/2} = e^{-\tau^2/t_{orbit}^2}$, so this sudden perturbation causes transitions with probabilities at the 0.1 % level, which we neglect. Thus we conclude that the wave function of the bound electron after the beta decay is the same as it was before the decay. The ground state of tritium (before the decay) is ($Z = 1$)

$$\psi_{T100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

The ground state of helium 3 (after the decay) is ($Z = 2$)

$$\psi_{He100}(r,\theta,\phi) = \frac{\sqrt{8}}{\sqrt{\pi a_0^3}} e^{-2r/a_0}$$

The wave function is unchanged in the decay, so the probability is found from the inner product of the two states:

$$\mathcal{P}_{T\to He} = \left|\langle \psi_{He100} | \psi_{T100} \rangle\right|^2$$

The inner product is

$$\begin{aligned}
\langle \psi_{He100} | \psi_{T100} \rangle &= \int \psi^*_{He100}(r,\theta,\phi)\psi_{T100}(r,\theta,\phi)\,dV \\
&= \int_0^\infty \int_0^{2\pi} \int_0^\pi \frac{\sqrt{8}}{\sqrt{\pi a_0^3}} e^{-2r/a_0} \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} r^2 \sin\theta \, d\theta \, d\phi \, dr \\
&= \frac{\sqrt{8}}{\pi a_0^3}\left\{\left(\int_0^\infty r^2 e^{-3r/a_0}\,dr\right)\left(\int_0^\pi \sin\theta\,d\theta\right)\left(\int_0^{2\pi} d\phi\right)\right\} \\
&= \frac{\sqrt{8}}{\pi a_0^3}\left(\frac{2a_0^3}{27}\right)(2)(2\pi) = \frac{16\sqrt{2}}{27}
\end{aligned}$$

giving a probability

$$\mathcal{P}_{T\to He} = \left|\tfrac{16\sqrt{2}}{27}\right|^2 = \tfrac{512}{729} \cong 0.702$$

For the second case, the final wave function is

$$\psi_{He16,3,0}(r,\theta,\phi) = R_{16,3}(r)Y_3^0(\theta,\phi)$$

The initial state is

$$\psi_{T100}(r,\theta,\phi) = R_{1,0}(r)Y_0^0(\theta,\phi)$$

The angular functions of these two wave functions are orthogonal, so the amplitude and hence probability is **zero**.

3. The perturbation is

$$H'(t) = V_0 x^2 e^{-t/\tau}$$

Time dependent perturbation theory tells us that the coefficient to be in a new state $|2\rangle$ at time T after starting in state $|1\rangle$ is :

$$c_2(T) = \frac{1}{i\hbar}\int_0^T \langle 2|H'(t')|1\rangle e^{i(E_2-E_1)t'/\hbar}\,dt'$$

For this case, we get

$$c_2(T) = \frac{1}{i\hbar}\int_0^T \langle 2|H'(t')|1\rangle e^{i(E_2-E_1)t'/\hbar}\,dt' = \frac{1}{i\hbar}\int_0^T \langle 2|V_0 x^2 e^{-t'/\tau}|1\rangle e^{i(E_2-E_1)t'/\hbar}\,dt'$$
$$= \frac{V_0}{i\hbar}\langle 2|x^2|1\rangle \int_0^T e^{-t'/\tau} e^{i(E_2-E_1)t'/\hbar}\,dt'$$

Let's tackle the spatial matrix element first

$$\langle 2|x^2|1\rangle = \int_0^L \varphi_2^*(x) x^2 \varphi_1(x)\,dx = \frac{2}{L}\int_0^L x^2 \sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}\,dx$$
$$= \frac{2}{L}\int_0^L x^2 \frac{1}{2}\left(\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)dx$$
$$= \frac{1}{\pi^3 L}\left[\begin{array}{l} 2\pi L^2 x\cos\frac{\pi x}{L} + \left(\pi^2 L x^2 - 2L^3\right)\sin\frac{\pi x}{L} \\ -\frac{1}{27}\left\{6\pi L^2 x\cos\frac{3\pi x}{L} + \left(9\pi^2 L x^2 - 2L^3\right)\sin\frac{3\pi x}{L}\right\}\end{array}\right]_0^L$$
$$= -\frac{16L^2}{9\pi^2}$$

Putting this into the equation above and doing the time integral, we get

$$c_2(T) = \frac{V_0}{i\hbar}\left(-\frac{16L^2}{9\pi^2}\right)\int_0^T e^{-t'/\tau} e^{i(E_2-E_1)t'/\hbar}\,dt' = -\frac{16L^2V_0}{i9\hbar\pi^2}\int_0^T e^{-t'/\tau}e^{i\omega_{21}t'}\,dt'$$
$$= -\frac{16L^2V_0}{i9\hbar\pi^2}\left[\frac{e^{i\omega_{21}t' - t'/\tau}}{i\omega_{21} - 1/\tau}\right]_0^T = -\frac{16L^2V_0}{i9\hbar\pi^2}\left[\frac{e^{i\omega_{21}T - T/\tau} - 1}{i\omega_{21} - 1/\tau}\right]$$

The unperturbed well energies are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \Rightarrow \quad \omega_{21} = \frac{E_2 - E_1}{\hbar} = \frac{3\pi^2\hbar}{2mL^2}$$

Putting this together we get

$$\mathcal{P}_{1->2}(T) = |c_2(T)|^2 = \left|-\frac{16L^2V_0}{i9\hbar\pi^2}\left[\frac{e^{i\omega_{21}T - T/\tau} - 1}{i\omega_{21} - 1/\tau}\right]\right|^2$$
$$= \left(\frac{16L^2V_0}{9\hbar\pi^2}\right)^2 \frac{1}{\omega_{21}^2 + 1/\tau^2}\left(e^{i\omega_{21}T - T/\tau} - 1\right)\left(e^{-i\omega_{21}T - T/\tau} - 1\right)$$
$$= \left(\frac{16L^2V_0}{9\hbar\pi^2}\right)^2 \frac{1}{\omega_{21}^2 + 1/\tau^2}\left[e^{-2T/\tau} + 1 - e^{-T/\tau}\left(e^{-i\omega_{21}T} + e^{i\omega_{21}T}\right)\right]$$
$$= \left(\frac{16L^2V_0}{9\hbar\pi^2}\right)^2 \frac{1}{\omega_{21}^2 + 1/\tau^2}\left[1 + e^{-2T/\tau} - 2e^{-T/\tau}\cos\omega_{21}T\right]$$

In the long time limit, we get the probability

$$\mathcal{P}_{1->2}(\infty)=\left(\frac{16L^2V_0}{9\hbar\pi^2}\right)^2\frac{1}{\omega_{21}^2+1/\tau^2}$$

19.3.3: The scattering potential is

$$V(r)=V_0e^{-r^2/r_0^2}$$

In the Born approximation, the scattering amplitude is

$$f(\theta,\phi)=-\frac{\mu}{2\pi\hbar^2}\int e^{-i\mathbf{q}\cdot\mathbf{r}}V(\mathbf{r})d^3\mathbf{r}$$

Thus we get

$$\begin{aligned}
f(\theta,\phi)&=-\frac{\mu}{2\pi\hbar^2}\int e^{-i\mathbf{q}\cdot\mathbf{r}}V_0e^{-r^2/r_0^2}d^3\mathbf{r}\\
&=-\frac{\mu}{2\pi\hbar^2}V_0\int_0^\infty e^{-r^2/r_0^2}r^2\,dr\int_0^{2\pi}d\phi\int_0^\pi \sin\theta e^{-iqr\cos\theta}\,d\theta\\
&=-\frac{\mu}{2\pi\hbar^2}V_0\int_0^\infty e^{-r^2/r_0^2}r^2\,dr[2\pi]\left[\frac{e^{-iqr\cos\theta}}{iqr}\right]_0^\pi\\
&=-\frac{2\mu}{\hbar^2}V_0\int_0^\infty e^{-r^2/r_0^2}r^2\frac{\sin qr}{qr}dr\\
&=-\frac{2\mu}{q\hbar^2}V_0\int_0^\infty e^{-r^2/r_0^2}r\sin qr\,dr
\end{aligned}$$

The radial integral can be looked up to give

$$\begin{aligned}
f(\theta,\phi)&=-\frac{2\mu}{q\hbar^2}V_0\frac{q\sqrt{\pi}r_0^3}{4}e^{-q^2r_0^2/4}\\
&=-\frac{\mu V_0}{\hbar^2}\frac{\sqrt{\pi}r_0^3}{2}e^{-q^2r_0^2/4}
\end{aligned}$$

The differential cross section is

$$\sigma(\theta,\phi)=|f(\theta,\phi)|^2=\left(\frac{\mu V_0r_0^2}{\hbar^2}\right)^2\frac{\pi r_0^2}{4}e^{-q^2r_0^2/2}$$

The total cross section is obtained by integrating over all detection angles, using $q^2=2k^2(1-\cos\theta)$

$$\sigma = \int \sigma(\theta,\phi)\,d\Omega = \int_0^{2\pi}\int_0^{\pi}\sigma(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi r_0^2}{4} e^{-k^2 r_0^2(1-\cos\theta)}\sin\theta\,d\theta\,d\phi$$

$$= \left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi r_0^2}{4} 2\pi \int_{-1}^{1} e^{-k^2 r_0^2(1-\cos\theta)}\,d(\cos\theta)$$

$$= \left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi r_0^2}{4} 2\pi e^{-k^2 r_0^2}\int_{-1}^{1} e^{xk^2 r_0^2}\,dx = \left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi r_0^2}{4} 2\pi e^{-k^2 r_0^2}\left[\frac{e^{xk^2 r_0^2}}{k^2 r_0^2}\right]_{-1}^{1}$$

$$= \left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi r_0^2}{4} 2\pi e^{-k^2 r_0^2}\frac{1}{k^2 r_0^2}\left(e^{k^2 r_0^2} - e^{-k^2 r_0^2}\right)$$

$$= \left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \frac{\pi^2}{2k^2}\left(1 - e^{-2k^2 r_0^2}\right)$$

19.5.4: Find the s-wave phase shift for a square well of depth V_0 and range r_0. The allowed wave functions inside and outside the square well are

$$R_\ell(r) = B_\ell\, j_\ell(\alpha r);\ r < r_0$$
$$R_\ell(r) = A_\ell\left[\cos\delta_\ell\, j_\ell(kr) - \sin\delta_\ell\,\eta_\ell(kr)\right];\ r > r_0$$

where (book uses k' for my α)

$$\alpha^2 = \frac{2\mu}{\hbar^2}(E + V_0) \qquad\qquad k^2 = \frac{2\mu}{\hbar^2}E$$

Match the wave functions and their derivatives at the well boundary:

$$B_\ell\, j_\ell(\alpha r_0) = A_\ell\left[\cos\delta_\ell\, j_\ell(kr_0) - \sin\delta_\ell\,\eta_\ell(kr_0)\right]$$
$$\alpha B_\ell\, j_\ell'(\alpha r_0) = kA_\ell\left[\cos\delta_\ell\, j_\ell'(kr_0) - \sin\delta_\ell\,\eta_\ell'(kr_0)\right]$$

Divide these two equations to eliminate the amplitudes in favor of the phase shifts.

$$\alpha\frac{j_\ell'(\alpha r_0)}{j_\ell(\alpha r_0)} = k\frac{\cos\delta_\ell\, j_\ell'(kr_0) - \sin\delta_\ell\,\eta_\ell'(kr_0)}{\cos\delta_\ell\, j_\ell(kr_0) - \sin\delta_\ell\,\eta_\ell(kr_0)}$$

Now focus on just the s-wave scattering, noting that

$$j_0(\rho) = \frac{\sin\rho}{\rho};\quad j_0'(\rho) = \frac{\cos\rho}{\rho} - \frac{\sin\rho}{\rho^2}$$
$$\eta_0(\rho) = -\frac{\cos\rho}{\rho};\quad \eta_0'(\rho) = \frac{\sin\rho}{\rho} + \frac{\cos\rho}{\rho^2}$$

Substitute

$$\alpha\frac{\dfrac{\cos\alpha r_0}{\alpha r_0}-\dfrac{\sin\alpha r_0}{(\alpha r_0)^2}}{\dfrac{\sin\alpha r_0}{\alpha r_0}}=k\frac{\cos\delta_0\left(\dfrac{\cos kr_0}{kr_0}-\dfrac{\sin kr_0}{(kr_0)^2}\right)-\sin\delta_0\left(\dfrac{\sin kr_0}{kr_0}+\dfrac{\cos kr_0}{(kr_0)^2}\right)}{\cos\delta_0\dfrac{\sin kr_0}{kr_0}+\sin\delta_0\dfrac{\cos kr_0}{kr_0}}$$

Rearrange

$$\alpha\left(\cot\alpha r_0-\frac{1}{\alpha r_0}\right)=k\frac{\dfrac{1}{kr_0}\cos(kr_0+\delta_0)-\dfrac{1}{(kr_0)^2}\sin(kr_0+\delta_0)}{\dfrac{1}{kr_0}\sin(kr_0+\delta_0)}$$

$$\alpha\left(\cot\alpha r_0-\frac{1}{\alpha r_0}\right)=k\left(\cot(kr_0+\delta_0)-\frac{1}{kr_0}\right)$$

$$\alpha\cot\alpha r_0=k\cot(kr_0+\delta_0)$$

Invert

$$\alpha\tan(kr_0+\delta_0)=k\tan\alpha r_0$$

$$kr_0+\delta_0=\tan^{-1}\left(\frac{k}{\alpha}\tan\alpha r_0\right)$$

$$\delta_0=-kr_0+\tan^{-1}\left(\frac{k}{\alpha}\tan\alpha r_0\right)$$

For small energy ($kr_0\ll 1$), we ignore the first term and get

$$\delta_0=\tan^{-1}\left(\frac{k}{\alpha}\tan\alpha r_0\right)$$

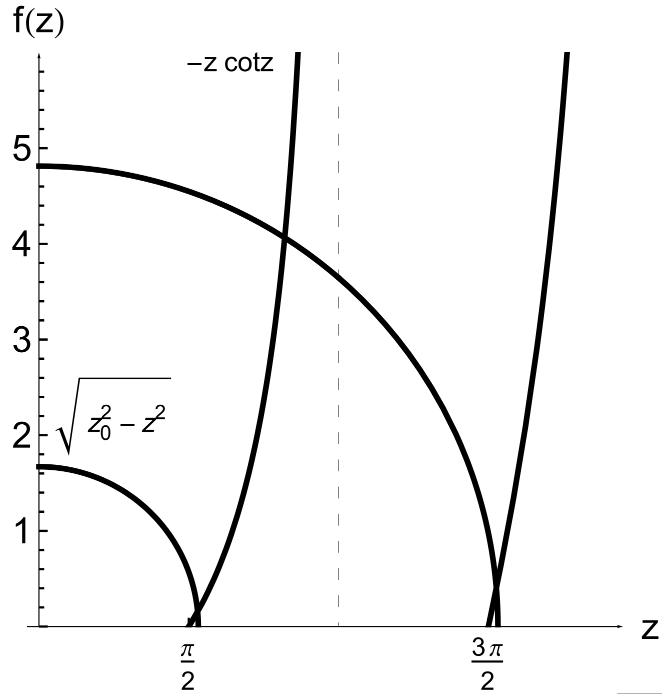
This will produce large scattering whenever $\tan\alpha r_0=\infty$, which gives $\delta_0=\pi/2$ and makes $\sigma=(4\pi/k^2)\sin^2\delta_0$ maximum. These resonances happen whenever

$$\alpha_n r_0=(2n-1)\frac{\pi}{2}$$

These same values of αr_0 satisfy the equation for bound states at zero energy (see Prob. 12.6.9):

$$\frac{\alpha}{k}=-\tan\alpha r_0 \quad\Rightarrow\quad -\alpha\cot\alpha r_0=0 \quad at\ \ k=0$$

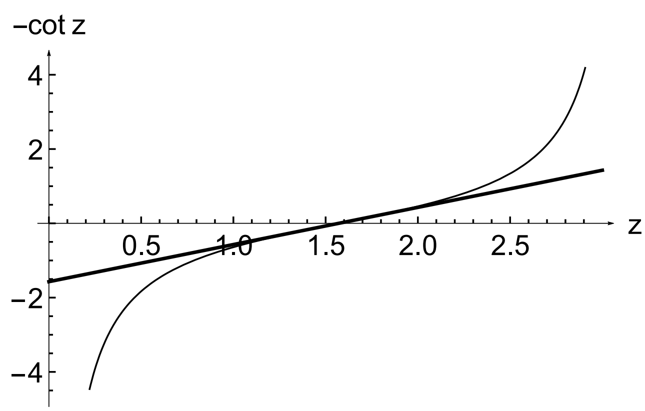
The resonances can be parameterized as in McIntyre Chap. 5 to produce plots like below (where we are using $z=\alpha r_0$).



The bound state energies occur when the circle of radius $z_0 = \sqrt{2\mu V_0 r_0^2/\hbar^2}$ intersects the $-z\cot z$ curves. For the zero energy resonances, this means we want the value of z_0 to be $\pi/2$, $3\pi/2$, *etc*. Each time the well depth (or width) increases to cause z_0 to equal one of these values, a new zero-energy resonance will occur.

To find how the resonances depend on energy (through $\alpha^2 = 2\mu\left(E+V_0\right)/\hbar^2$), note that near the zero-energy resonances values, the $-\cot z$ function is linear in $z = \alpha r_0$:

$$\cot\alpha r_0 = -\alpha r_0 + \alpha_n r_0 = -\alpha r_0 + \left(2n-1\right)\frac{\pi}{2}$$



Hence the phase shift becomes (we set $k = k_n$ as the energy dependence is primarily in the cot function)

$$\delta_0 = \tan^{-1}\left(\frac{k_n}{\alpha}\tan\alpha r_0\right) = \tan^{-1}\left(\frac{k_n}{\alpha\cot\alpha r_0}\right)$$

$$= \tan^{-1}\left(\frac{k_n}{\alpha\left[-\alpha r_0 + \alpha_n r_0\right]}\right) = \tan^{-1}\left(\frac{k_n}{\left(\alpha\alpha_n - \alpha^2\right)r_0}\right)$$

Since these are zero energy resonances, we have $\alpha_n = \sqrt{2\mu V_0/\hbar^2}$, giving

$$\delta_0 = \tan^{-1}\left(\frac{k_n}{r_0\left(\sqrt{\frac{2\mu}{\hbar^2}\left(E+V_0\right)}\sqrt{\frac{2\mu V_0}{\hbar^2}} - \frac{2\mu}{\hbar^2}\left(E+V_0\right)\right)}\right)$$

$$= \tan^{-1}\left(\frac{k_n}{\frac{2\mu r_0}{\hbar^2}\left(\sqrt{V_0\left(E+V_0\right)} - \left(E+V_0\right)\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\hbar^2 k_n}{\mu r_0}}{2V_0\left(\sqrt{1+\frac{E}{V_0}} - \left(1+\frac{E}{V_0}\right)\right)}\right)$$

Now assume $E \ll V_0$ and expand

$$\delta_0 \cong \tan^{-1}\left(\frac{\frac{\hbar^2 k_n}{\mu r_0}}{2V_0\left(1+\frac{E}{2V_0} - \left(1+\frac{E}{V_0}\right)\right)}\right)$$

$$\cong \tan^{-1}\left(\frac{\frac{\hbar^2 k_n}{\mu r_0}}{2V_0\left(0-\frac{E}{2V_0}\right)}\right) \cong \tan^{-1}\left(\frac{\frac{\hbar^2 k_n}{\mu r_0}}{0-E}\right)$$

This has the desired resonance form

$$\delta_0 = \delta_b + \tan^{-1}\left(\frac{\Gamma/2}{E_0 - E}\right)$$

with $\delta_b = 0$, $E_0 = 0$ and $\Gamma/2 = \hbar^2 k_n/\mu r_0$