PH 651 Quantum Mechanics 21 October 2015

Homework 3

Due Wednesday 4 November

1. 5.2.1, p. 163 from Shankar

2. 5.2.3, p. 163 from Shankar

3. Consider a system with a three-dimensional state space spanned by the orthonormal basis $|1\rangle, |2\rangle, |3\rangle$. In this basis, the Hamiltonian *H* and two observables *A* and *B* are represented by

$$H \doteq \hbar\omega_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}; \quad A \doteq a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \quad B \doteq b \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where ω_0, *a* and *b* are positive real constants. At time *t* = 0, the system is in the state

$$|\psi(0)\rangle = \tfrac{1}{\sqrt{2}}|1\rangle + \tfrac{1}{2}|2\rangle + \tfrac{1}{2}|3\rangle$$

a) At time *t* = 0, the energy of the system is measured. What values are possible and what are the probabilities of their measurement? Calculate the expectation value and the uncertainty of the energy (measured at *t* = 0).

b) Calculate the state vector $|\psi(t)\rangle$ at a later time *t*, assuming no energy measurement at *t* = 0.

c) Calculate the expectation values of the observables *A* and *B* at time *t*. Explain your results.

4. A free particle has an initial wave function that can be expressed as

$$\psi(x,0) = A\int_{-\infty}^{\infty} e^{-|p|/p_0}\ e^{ipx/\hbar}\ dp$$

where A and p_0 are constants.

a) What is the probability that a measurement of the momentum at time $t = 0$ yields a result in the range $-p_1$ to p_1? Sketch the probability as a function of the parameter p_1.

b) Calculate the wave function at time $t = 0$ in the position representation (i.e. X basis). Sketch the wave function.

c) How is the momentum measurement probability from part (a) changed if performed at a later time t?

5. Consider a general quantum state vector $|\psi\rangle$.

a) Express the spatial wave function $\psi(x)$ in bra-ket notation.

b) Express the momentum space wave function $\psi(p)$ in bra-ket notation.

c) Express the quantum state vector in matrix notation.

d) Express the spatial wave function in terms of the momentum space wave function, using both bra-ket notation and wave function notation.

e) Express the spatial probability density $\mathcal{P}(x)$ in bra-ket notation and in wave function notation.

f) Express the momentum space probability density $\mathcal{P}(p)$ in bra-ket notation and in wave function notation.

g) Express the probability $\mathcal{P}(a \le x \le b)$ that a particle is found in the region $a \le x \le b$ in wave function notation.

h) Express the probability $\mathcal{P}(p_1 \le p \le p_2)$ that a particle is found with momentum $p_1 \le p \le p_2$ in wave function notation.

i) Express the state vector normalization condition in bra-ket notation, in wave function notation, and in matrix notation.

j) Express the inner product between two states $|\psi\rangle$ and $|\phi\rangle$ in bra-ket notation, in wave function notation, and in matrix notation.

k) Express the probability $\mathcal{P}(E_n)$ that a particle is found with energy E_n in bra-ket notation, in wave function notation, and in matrix notation.