PH 632: Electromagnetic Theory

Homework 6

Due 3/11/11

1) Jackson 7.22
2) Griffiths 9.37
3) Brau 7.3

Griffiths 9.37

Problem 9.37 According to Snell's law, when light passes from an optically dense medium into a less dense one ($n_1 > n_2$) the propagation vector **k** bends *away* from the normal (Fig. 9.28). In particular, if the light is incident at the **critical angle**

$$\theta_c \equiv \sin^{-1}(n_2/n_1), \tag{9.200}$$

then $\theta_T = 90°$, and the transmitted ray just grazes the surface. If θ_I *exceeds* θ_c, there is no refracted ray at all, only a reflected one (this is the phenomenon of **total internal reflection**, on which light pipes and fiber optics are based). But the *fields* are not zero in medium 2; what we get is a so-called **evanescent wave**, which is rapidly attenuated and transports no energy into medium 2.[19]

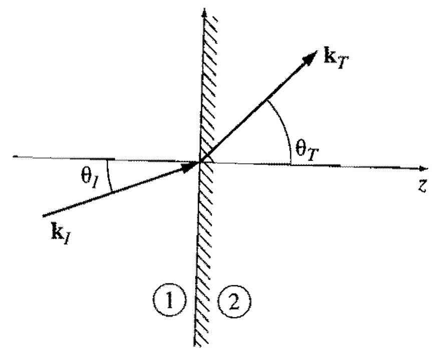

Figure 9.28

A quick way to construct the evanescent wave is simply to quote the results of Sect. 9.3.3, with $k_T = \omega n_2/c$ and

$$\mathbf{k}_T = k_T(\sin\theta_T\,\hat{\mathbf{x}} + \cos\theta_T\,\hat{\mathbf{z}});$$

the only change is that

$$\sin\theta_T = \frac{n_1}{n_2}\sin\theta_I$$

is now greater than 1, and

$$\cos\theta_T = \sqrt{1-\sin^2\theta_T} = i\sqrt{\sin^2\theta_T - 1}$$

is imaginary. (Obviously, θ_T can no longer be interpreted as an *angle*!)

(a) Show that

$$\tilde{\mathbf{E}}_T(\mathbf{r},t) = \tilde{\mathbf{E}}_{0_T} e^{-\kappa z} e^{i(kx-\omega t)}, \tag{9.201}$$

where

$$\kappa \equiv \frac{\omega}{c}\sqrt{(n_1\sin\theta_I)^2 - n_2^2} \quad\text{and}\quad k \equiv \frac{\omega n_1}{c}\sin\theta_I. \tag{9.202}$$

This is a wave propagating in the x direction (*parallel* to the interface!), and attenuated in the z direction.

(b) Noting that α (Eq. 9.108) is now imaginary, use Eq. 9.109 to calculate the reflection coefficient for polarization parallel to the plane of incidence. [Notice that you get 100% reflection, which is better than at a conducting surface (see, for example, Prob. 9.21).]

(c) Do the same for polarization perpendicular to the plane of incidence (use the results of Prob. 9.16).

(d) In the case of polarization perpendicular to the plane of incidence, show that the (real) evanescent fields are

$$\left.\begin{aligned} \mathbf{E}(\mathbf{r},t) &= E_0 e^{-\kappa z}\cos(kx-\omega t)\,\hat{\mathbf{y}}, \\ \mathbf{B}(\mathbf{r},t) &= \frac{E_0}{\omega}e^{-\kappa z}\left[\kappa\sin(kx-\omega t)\,\hat{\mathbf{x}} + k\cos(kx-\omega t)\,\hat{\mathbf{z}}\right]. \end{aligned}\right\} \tag{9.203}$$

(e) Check that the fields in (d) satisfy all of Maxwell's equations (9.67).

(f) For the fields in (d), construct the Poynting vector, and show that, on average, no energy is transmitted in the z direction.

[19] The evanescent fields can be detected by placing a second interface a short distance to the right of the first; in a close analog to quantum mechanical **tunneling**, the wave crosses the gap and reassembles to the right. See F. Albiol, S. Navas, and M. V. Andres, *Am. J. Phys.* **61**, 165 (1993).

Brau 7.3

EXERCISE 7.3

Consider an optical medium that is transparent except for a finite number of very narrow absorption lines at wavelengths λ_i. Use the Kramers–Kronig relations to show that the index of refraction at a wavelength λ between the absorption lines is given by the expression

$$n^2 - 1 = \sum_i \frac{A_i\lambda^2}{\lambda^2 - \lambda_i^2} \tag{7.86}$$

for some constants A_i. This expression is frequently used as an analytical fit to the experimentally measured index of refraction of various substances. The coefficients A_i and λ_i are called the Sellmeier coefficients.