Ph 451 Quantum Mechanics Capstone Winter 2012

FINAL EXAM – March 21, 2012

This exam is closed book and closed notes except for the information on this cover sheet. Please do all your work in the blue books. **Only the blue books will be graded!**

There are a total of 200 points possible on this exam.

Budget your time wisely! Not all questions are of equal difficulty.

Equations and integrals that may be useful:

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 1 - 2\sin^2\theta \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\int_0^\infty e^{-a^2x^2}dx = \frac{1}{2a}\sqrt{\pi} \qquad \int_0^\infty xe^{-x^2}dx = \frac{1}{2} \qquad \int_0^\infty x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{4}$$

$$\int \sin mx \sin nx\, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)},\ (m^2 \neq n^2)$$

$$\int \cos mx \cos nx\, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)},\ (m^2 \neq n^2)$$

$$\int \sin mx \cos nx\, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)},\ (m^2 \neq n^2)$$

$$\langle\psi|\hat{A}^\dagger|\phi\rangle = \langle\phi|\hat{A}|\psi\rangle^* \qquad \sum_n |n\rangle\langle n| = 1 \qquad \int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$$

$$E_n = n^2\frac{\hbar^2\pi^2}{2ma^2} \qquad \varphi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

$$V(x) = \tfrac{1}{2}m\omega^2x^2 \qquad a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - i\frac{p}{m\omega}\right)$$

$$a|n\rangle = \sqrt{n}|n-1\rangle \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\varphi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \qquad \varphi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{m\omega}{\hbar}}\frac{1}{\sqrt{2}}2xe^{-\frac{m\omega}{2\hbar}x^2}$$

$$\varphi_n(x) = \frac{1}{2^{n/2}}\frac{1}{\sqrt{n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2}$$

$$E_n^{(1)} = \left\langle n^{(0)}\middle|H'\middle|n^{(0)}\right\rangle \qquad E_n^{(2)} = \sum_{m\neq n}\frac{\left|\left\langle n^{(0)}\middle|H'\middle|m^{(0)}\right\rangle\right|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\left|n^{(1)}\right\rangle = \sum_{m\neq n} c_{nm}^{(1)}\left|m^{(0)}\right\rangle \qquad c_{nm}^{(1)} = \frac{\left\langle m^{(0)}\middle|H'\middle|n^{(0)}\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

$$\left[J_x, J_y\right] = i\hbar J_z \qquad J_\pm = J_x \pm iJ_y$$

$$J_\pm\left|j,m\right\rangle = \hbar\left[j\left(j+1\right) - m\left(m\pm1\right)\right]^{\frac{1}{2}}\left|j,m\pm1\right\rangle \qquad \left|JM\right\rangle = \sum_{m_1 m_2}\left|j_1 j_2 m_1 m_2\right\rangle\left\langle j_1 j_2 m_1 m_2\middle|JM\right\rangle$$

$$c_f\left(t\right) = \frac{1}{i\hbar}\int_0^t \left\langle f\middle|H'\left(t'\right)\middle|i\right\rangle e^{i\frac{E_f - E_i}{\hbar}t'}\,dt'$$