Ph 435 Classical Mechanics Capstone Spring 2010

FINAL EXAM – June 8, 2010

This exam is closed book and closed notes except for the information on this cover sheet. Please do all your work in the blue books. **Only the blue books will be graded!**

There are 4 main parts with a total of 100 points possible on this exam.

Budget your time wisely! Not all questions are of equal difficulty.

Equations and integrals that may be useful:

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 1 - 2\sin^2\theta \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{P}}}{dt} \qquad dp = mdv + udm$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| \qquad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}\sin\theta\, d\theta$$

$$L = T - U \qquad p_j = \frac{\partial L}{\partial \dot{q}_j} \qquad H = \sum_{j=1}^{s} p_j \dot{q}_j - L$$

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 \qquad f_k(q_1, q_2, \ldots q_s, t) = 0 \qquad \frac{\partial L}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) + \sum_{k=1}^{m} \lambda_k(t)\frac{\partial f_k}{\partial q_j} = 0$$

$$\dot{q}_j = \frac{\partial H}{\partial p_j} \qquad \dot{p}_j = -\frac{\partial H}{\partial q_j} \qquad \frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$T = \frac{1}{2}\sum_{jk} M_{jk}\dot{q}_j\dot{q}_k \qquad U = \frac{1}{2}\sum_{jk} K_{jk} q_j q_k \qquad \left|\underline{\mathbf{K}} - \omega^2 \underline{\mathbf{M}}\right| = 0$$

$$\left(\underline{\mathbf{K}} - \omega_r^2 \underline{\mathbf{M}}\right)\vec{\mathbf{a}}_r = 0 \qquad \sum_j (K_{kj} - M_{kj}\omega_r^2) a_{jr} = 0 \qquad \underline{\mathbf{a}}^T \underline{\mathbf{M}}\underline{\mathbf{a}} = \underline{\mathbf{1}}$$

$$\sum_{jk} a_{jr} M_{jk} a_{ks} = \delta_{rs} \qquad q_j = \sum_r a_{jr}\xi_r \qquad \vec{\mathbf{q}} = \underline{\mathbf{a}}\vec{\xi} \qquad \vec{\xi} = \underline{\mathbf{a}}^T \underline{\mathbf{M}}\vec{\mathbf{q}}$$

$$\int \frac{dx}{1+e^x} = x - \ln\left(1+e^x\right) \qquad \int \frac{dx}{a+bx} = \frac{1}{b}\ln\left(a+bx\right) \qquad \int \frac{xdx}{a^2+x^2} = \frac{1}{2}\ln\left(a^2+x^2\right)$$

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1) \qquad \int \ln(a+bx)dx = \frac{a+bx}{b}\ln(a+bx) - \frac{a+bx}{b}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \qquad \ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \ldots$$