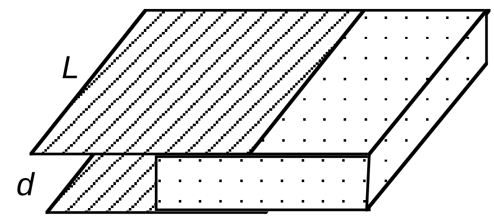
THIS IS A CLOSED BOOK EXAMINATION. You may use not any other written materials except one formula sheet you have brought with you written in your own hand, which must be turned in together with the examination paper at the end of the examination. You may not communicate with anyone except the examiner about the examination until you have turned it in.
Show all your work. Partial credit will be given for answers which are not completely correct. On the other hand an answer will not be acceptable, even if correct, unless you explain how you arrived at the answer. Be sure to put your name on your paper before you hand it in! Remember that you may express the answer to a later part of a problem in terms of the unknown answer to an earlier part of the problem.

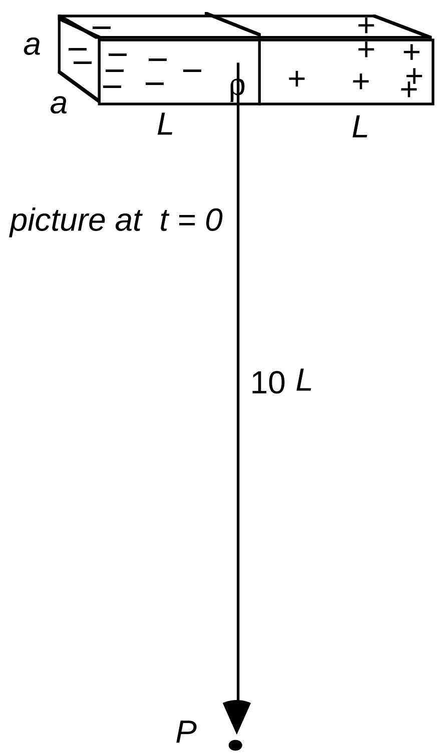
Problem 1 A capacitor consists of two square conducting plates, each of size $L \times L$, separated by a distance $d << L$. A square block of insulating dielectric of the same dimensions is inserted halfway so that it fills the right half of the space between the plates. The plates are then connected to a battery of voltage V .



Find the force on the dielectric block. Give magnitude and direction..

Problem 2 A rectangular box of length $2L$ and square cross section $a \times a$, with its square ends at $x = \pm L$, contains a charge density $\rho = \rho_0\, x/L$ where x is the distance from the midplane. The charge density does not change for times $t < 0$.

a. Find the dipole moment of the charge distribution, $\boldsymbol{p}_0$ = ? Give magnitude and direction.

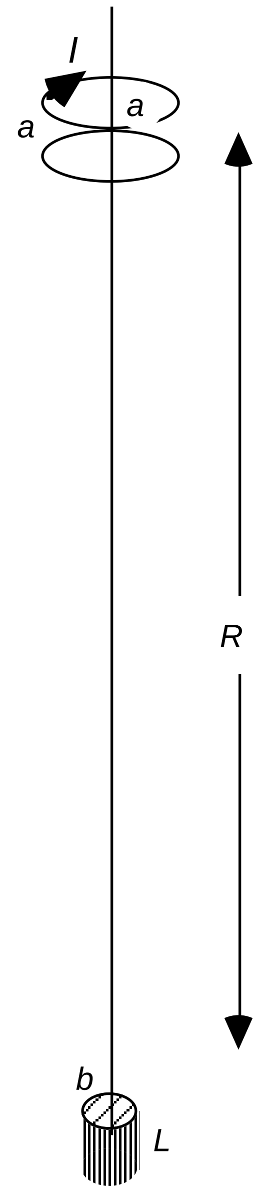


At time $t = 0$, find the electric field at a point P located a distance $D = 10\,L$ from the center of the box in its midplane.

b. Starting at time $t = 0$, the box's charge density oscillates with time, so that $\rho(x, t) = \rho_0 \frac{x}{L} \cos(\omega t)$. Find the current density inside the box, $J(x, t)$ = ?

Problem 3 Two circular loops of radius a lie in two horizontal planes separated by a distance a. Each loop carries current I in a clockwise direction seen from above.

a. Find the **direction** of the magnetic field $\boldsymbol{B}$ at a point which lies a distance $R \gg a$ away from the midplane of the loops, and directly below them.



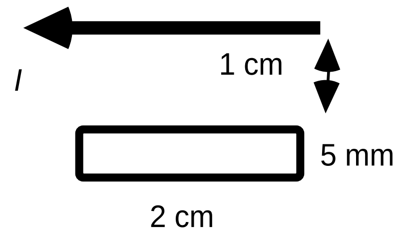
Show that its (approximate) **magnitude** is $B \approx \dfrac{\mu_o I a^2}{R^3}$.

b. A cylinder of length L and radius b is placed at the point specified in part **a**. It is made of magnetic material of permeability $\mu = 9\,\mu_o$. Find the direction and magnitude of the magnetic field inside the cylinder.

c. Find the distribution of the effective induced current in the cylinder.

d. Find the force $\boldsymbol{F}$ on the cylinder. Give direction and magnitude.

Problem 4.



A long straight thin wire carries an alternating current $I(t) = I_0 \sin(\omega t)$ parallel to the length of a paper clip made of conducting material.
The paper clip is 2 cm long and 1/2 cm across.
It lies in a plane with the wire, 1 cm away from it.
The amplitude of the current is $I_0 = 10$ amperes and its frequency is $f = 60$ hertz.

a. In what direction is the emf at time $t = 0$?

Find the emf induced in the paper clip, $V_{emf}(t) = ?$.

b. Find the mutual inductance of the wire and clip, $M_{wc} = ?$

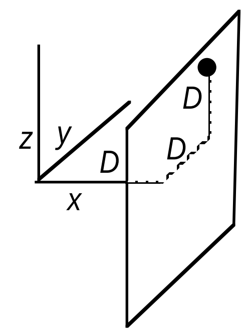
Problem 5

A short antenna emits a spherical wave, which is measured very far from the antenna as

$$\boldsymbol{E}(\boldsymbol{r},t) = V_0 \frac{\sin\theta}{r} \cos(kr-\omega t)\ \hat{\theta}\ , \text{ with } \frac{\omega}{k} = c\ .$$

a. Estimate the dipole moment of the antenna, $\boldsymbol{p}(t)$ = ?

The wave strikes a non-conducting windowpane with index of refraction n (assume permeability μ_0) which is located in a plane at $x = D$.



Assume that $kD >> 1$, computing your answer to leading order in powers of D^{-1}.

b. At a position $(x, y, z) = (D, D, D)$. find and give in the Cartesian coordinate system:

i. A unit vector normal to the pane, $\hat{\boldsymbol{n}}$ =

ii. A unit vector in the direction of propagation of the incident wave, $\hat{\boldsymbol{k}}_{inc}$ =

iii. A unit vector normal to the plane of incidence, $\hat{\boldsymbol{u}}_{\perp}$ =

iv. A unit vector in the direction of propagation of the reflected wave, $\hat{\boldsymbol{k}}_{refl}$ =

v. The sine and cosine of the angle of incidence, $\sin\theta_I$ = , $\cos\theta_I$ =

vi. The sine and cosine of the angle of reflection, $\sin\theta_R$ = , $\cos\theta_R$ =

vii. A unit vector along the incident electric field direction, $\hat{\boldsymbol{\varepsilon}}$ =

viii. The sine and cosine of the polarization angle between the incident electric field and the plane of polarization, $\sin\alpha$ = $\cos\alpha$ =

REMEMBER: YOU MAY EXPRESS THE ANSWER TO ANY PART OF A PROBLEM IN TERMS OF THE ANSWERS TO PREVIOUS PARTS

PROBLEM 5 CONTINUED ON NEXT PAGE

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Problew 5 CONTINUED

c. At the same position, $(x, y, z) = (D, D, D)$, find:

i. The intensity of the incident wave, $I_{\text{inc}} =$

ii. The intensity of the transmitted wave, $I_{\text{trans}} =$

iii. The fractional polarization of the transmitted wave, $P \equiv \dfrac{I_{||} - I_{\perp}}{I_{||} + I_{\perp}} =$

END OF EXAMINATION