More on Conducting Sphere

Griffiths 3.7   In Example 3.2 we assumed that the conducting sphere was grounded ($V = 0$). But with the addition of a second image charge, the same basic model will handle the case of a sphere at any given potential V_O (relative, of course, to zero at infinity). What charge should you use, and where should you put it?

Find the force of attraction between a point charge q and a neutral conducting sphere .

solution: courtesy D.S. Matusevich
Because of the rotational symmetry about the axis
        (defined by the original charge and the center of the sphere)
            all image charges have to lie on this line.

The book shows where to place the first image charge, which (together with the real charge) makes the sphere at radius R an equipotential.

By the superposition principle, we can just add another charge to adjust the potential of the sphere without disturbing the original result.

A point charge q'' at the sphere's center will do the job:
        its equipotentials are spheres with the same origin.
        At radius R the potential of q'' is $V''(\mathbf{R}) = \dfrac{q''}{4\pi\epsilon_o R} = V_O,$

            from which $q'' = 4\pi\epsilon_o R\, V_O$.

Gauss' law implies that the total charge on the sphere is $q' + q''$;
                        to be neutral, then, $q'' = -q' = \dfrac{R}{a}q$

Superposition also applies to the forces,

        so to the force of the original distribution, eq. 3.18 ,
            we add the new force:

$$F = -\frac{q^2 Ra}{4\pi\epsilon_0 (a^2 - R^2)^2} + \frac{q\,q'}{4\pi\epsilon_0 a^2} = -\frac{q^2 R^3}{4\pi\epsilon_0 a(a^2 - R^2)^2}\left(2 - \frac{R^2}{a^2}\right)$$