Radiation Pressure

Calculate the average radiation pressure on an ideal conductor for a wave in air of amplitude E_0 and frequency ω incident at angle θ. Account for this pressure (qualitatively) in terms of the forces on free electrons in the conductor.

(*HINT*: first find the incident wave's momentum and that of the reflected wave. How much momentum is delivered to the conductor in a time Δt ?)

The momentum density of the wave is $\boldsymbol{S}/c^2 = \boldsymbol{E} \times \boldsymbol{H}/c^2$

where $\boldsymbol{E} = \boldsymbol{E}_0 \cos(\omega t - \boldsymbol{k} \bullet \boldsymbol{r})$, $\boldsymbol{H} = \frac{-1}{\omega\mu} \boldsymbol{k} \times \boldsymbol{E}$ $\quad \frac{\boldsymbol{S}}{c^2} = \frac{E^2}{\mu c^3} \hat{\boldsymbol{k}}$.

The average value of E^2 is $\frac{1}{2} E_0{}^2$.

The amount of momentum $\Delta \boldsymbol{p}$ going through an area $\Delta \boldsymbol{A}$ in time Δt is the amount in a length $c\, \Delta t$ and cross section area $\Delta \boldsymbol{A} \bullet \hat{\boldsymbol{k}}$,

$$|\Delta \boldsymbol{p}\,|_{\text{av}} = \frac{\boldsymbol{S}_{\text{av}}}{c^2} \bullet \Delta \boldsymbol{A}\, c \Delta t = \frac{E_0{}^2}{2\mu c^2} \Delta A \;\Delta t \;\cos\theta,$$

where θ = angle of incidence = angle between normal and $\boldsymbol{k}$.

The direction of $\Delta \boldsymbol{p}$ is along $\boldsymbol{k}$.

From the arriving momentum of the incident wave,
we have to subtract the departing momentum of the reflected wave.
The components of $\Delta \boldsymbol{p}$ parallel to the surface are equal, so they cancel;
the perpendicular components, $|\Delta \boldsymbol{p}| \cos\theta$, are also equal but add together,

so $$\Delta p_{\text{tot}} = 2\, |\Delta \boldsymbol{p}|_{\text{av}} \cos\theta = \frac{E_0{}^2}{\mu c^{\,2}} \Delta A \;\Delta t \;\cos^2\theta .$$

The force on the area is $F = \Delta p_{\text{tot}} / \Delta t$,

and the pressure is $$P = F/\Delta A = \frac{\Delta p_{\text{tot}}}{\Delta t \;\Delta A} = \frac{E_0{}^2}{\mu c^{\,2}} \cos^2\theta .$$

The part of $\boldsymbol{E}_0$ to the surface pushes the electrons in and out;
the part of $\boldsymbol{E}_0$ || to the surface causes surface currents
` which get a push from the magnetic field of the wave.
This problem is a modified version of Problem 8.23 in Griffiths' 2nd ed.