COULOMB SOLVES LAPLACE

LCL 11-7 More solutions of Laplace's equation

a. $\nabla^2 (1/r) = 0$:

work in either cartesian or spherical coordinates.

cartesian: $\nabla^2 (1/r) = \left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} \right)(x^2 + y^2 + z^2)^{-1/2}$

$$\frac{\partial^2}{\partial x^2}(x^2+y^2+z^2)^{-1/2} =$$

$$-\frac{\partial}{\partial x} \frac{x}{(x^2+y^2+z^2)^{3/2}} = 3\frac{x^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}}$$

similarly for $\dfrac{\partial^2}{\partial y^2}, \dfrac{\partial^2}{\partial z^2} \Rightarrow$

$$\nabla^2 (1/r) = 3\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{5/2}} - 3\frac{1}{(x^2+y^2+z^2)^{3/2}} = 0.$$

b. $\nabla^2 \dfrac{\partial}{\partial x}(1/r) = \dfrac{\partial}{\partial x} \nabla^2 (1/r) = 0;$

similarly for $\dfrac{\partial^2}{\partial x^2}(1/r)$, $\dfrac{\partial^2}{\partial x \partial y}(1/r)$