

LAPLACE EQN CARTESIAN EXAMPLE

LCL 11-6 More solutions of Laplace's equation

look for solutions of form $V(x) = (X(x) + Y(y) + Z(z))$

$$\nabla^2 (X(x) + Y(y) + Z(z)) = \frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} + \frac{\partial^2 Z}{\partial z^2} = 0$$

each term depends on different variable \Rightarrow
each has to be constant

$$\frac{\partial^2 X}{\partial x^2} = C_x, \quad \frac{\partial^2 Y}{\partial y^2} = C_y, \quad \frac{\partial^2 Z}{\partial z^2} = C_z, \quad C_x + C_y + C_z = 0.$$

then $X = \frac{1}{2} C_x x^2 + A_x x + B_x$, etc. \Rightarrow

solutions of form $V(x) = C_x x^2 + C_y y^2 + C_z z^2$
 $+ A_x x + A_y y + A_z z + B$

satisfies $\nabla^2 V = 0$ iff $C_x + C_y + C_z = 0$.