LCL 20-7 The iron pipe

A wire carrying current I_{free} runs along the axis of an iron pipe.

Cylindrical symmetry:

translational along axis, rotational around axis $\Rightarrow$ $\boldsymbol{H} = H(\rho)\,\hat{\phi}$

use integral form of Ampere's law:

$$\oint \boldsymbol{H} \cdot d\boldsymbol{L} = \int \boldsymbol{J} \cdot dA = I_{free} \quad \text{the free current enclosed}$$

azimuthal path at fixed z, $\rho \Rightarrow d\boldsymbol{L} = \hat{\phi}\rho\, d\phi$,

$$\oint \boldsymbol{H} \cdot d\boldsymbol{L} = \int_0^{2\pi} d\phi\, \rho H(\rho) = 2\pi\, \rho H(\rho) = I_{free}$$

in all regions,

$$\boldsymbol{H}(\rho) = \frac{I_{free}}{2\pi\rho}\,\hat{\phi},\ \boldsymbol{B}(\rho) = \mu \boldsymbol{H}(\rho)\ ,\ \boldsymbol{M}(\rho) = \boldsymbol{B}(\rho)/\mu_o - \boldsymbol{H}(\rho) = \boldsymbol{H}(\rho)\,(1 - \mu/\mu_o)$$

where $\mu = \mu(\rho) = \mu_o$ except in iron $\Rightarrow$

$\boldsymbol{M} = 0$ except in iron where $M(\rho) = (\mu/\mu_o - 1)\dfrac{I_{free}}{2\pi\rho}$

No current inside iron: $\boldsymbol{J}_{eff} = \nabla \times \boldsymbol{M} = (\mu/\mu_o - 1)(-\hat{\rho}\dfrac{\partial}{\partial z} + \dfrac{1}{\rho}\dfrac{\partial}{\partial \rho}\rho\,)\dfrac{I_{free}}{2\pi\rho} = 0$

The induced (effective) current density at the surface of the iron is

$\alpha_{eff} = \boldsymbol{M} \times \hat{n}$ where $\hat{n}$ is the normal to the surface.

$\boldsymbol{M}$ points along $\boldsymbol{B}$ and $\boldsymbol{H}$,

on the inner surface of the pipe,

α_{eff} flows in the same direction as the original current

on the outer surface, α_{eff} flows against it.

Total current on each surface is $(\mu/\mu_o - 1)I_{free}$,

This result can be seen immediately from $\oint \boldsymbol{B} \cdot d\boldsymbol{L} = I_{eff}$