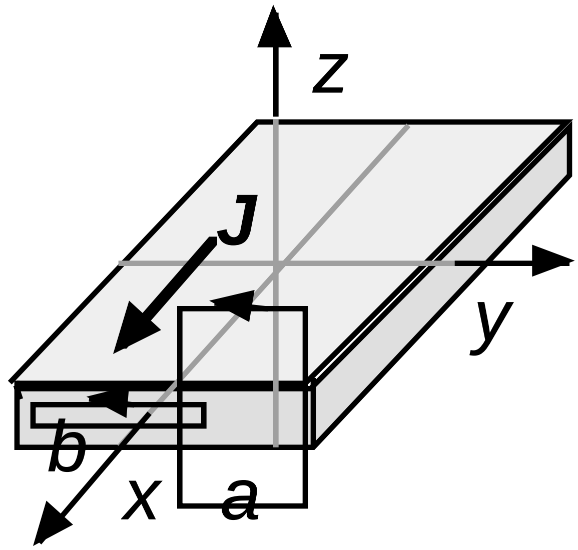
P. Siemens

Slab of Current



Griffiths 5.14

A thick slab extending

from $z = -a$ to $z = +a$

carries a uniform volume current

$$\mathbf{J} = J\hat{x} \; .$$

FInd the magnetic field both inside and outside the slab.

<u>solution:</u>

symmetry of problem, together with right-hand rule, implies

- **B** is parallel to the y-axis
- **B** is independent of x and y .
- **B** is reflection-antisymmetric about xy plane.

conclude: $\mathbf{B} = \hat{y}\; B(z)$ where $B(-z) = -B(z)$.

Apply Ampere's integral law to rectangular paths in xz plane, placed symmetrically about the plane $z = 0$.

<u>case</u> a : ends of rectangle lie outside slab, then

$$\oint \mathbf{B} \bullet d\mathbf{L} = -B(z)\,\Delta y + 0 + B(-z)\,\Delta y + 0 = -2B(z)\,\Delta y$$

total current through rectangle $= \int \mathbf{J} \bullet d\,area = J\,(2a\,\Delta y)$

equate: $B(z) = -J\,a$ for z outside slab, in y-direction

<u>case</u> b : ends of rectangle lie inside slab, then

$$\oint \mathbf{B} \bullet d\mathbf{L} = -B(z)\,\Delta y + 0 + B(-z)\,\Delta y + 0 = -2B(z)\,\Delta y$$

total current through rectangle $= \int \mathbf{J} \bullet d\,area = J\,(2z\;\Delta y)$

equate: $B(z) = -J\,z$ for z inside slab, in y-direction

<u>check:</u> $\bullet\mathbf{B} = 0$ everywhere since $\frac{\partial B_y}{\partial y} = 0.$

outside slab: $\times\,\mathbf{B} = 0$, since **B** is constant

inside slab: $\times\,\mathbf{B} = -\hat{x}\,\frac{\partial B_y}{\partial z} = \hat{x}\,J$ √