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MAGNETIC DISC

A disc of iron of radius *a* and thickness *s* is magnetized parallel to its axis. Calculate *B* on the axis, outside the iron.

Biot-Savart law: $$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3\mathbf{R}\,\frac{\mathbf{J}(\mathbf{r})\times(\mathbf{r}-\mathbf{R})}{|\mathbf{r}-\mathbf{R}|^3}$$

$\mathbf{J}\, d^3\mathbf{r} \to \alpha\, d^2S$, $\alpha = \mathbf{M}\times\hat{\mathbf{n}}$, $\hat{\mathbf{n}}$ = normal to surface

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\oint dS\,\frac{\alpha\times(\mathbf{r}-\mathbf{R})}{|\mathbf{r}-\mathbf{R}|^3}$$

integrand vanishes on top and bottom surfaces, where $\mathbf{M}\parallel\hat{\mathbf{n}}$

sides: $dS = dz\,\rho\, d\phi$, $\alpha = M\,\hat{\mathbf{z}}\times\hat{\rho} = M\hat{\phi}$

$$\alpha\times(\mathbf{r}-\mathbf{R}) = M\hat{\phi}\times[a\hat{\rho} + (z-Z)\,\hat{\mathbf{z}}]$$
$$= M\,[a(-\hat{\mathbf{z}}) + (z-Z)(-\hat{\rho})]$$

$$|\mathbf{r}-\mathbf{R}|^2 = (z-Z)^2 + a^2$$

$$\mathbf{B}(0,z) = \frac{\mu_0 M}{4\pi}\int_0^{2\pi} a\,d\phi\int_{-s/2}^{s/2} dZ\,\frac{a\,(-\hat{\mathbf{z}}) + (z-Z)(-\hat{\rho})}{((z-Z)^2 + a^2)^{3/2}}$$

the radial part along $\hat{\rho}$ averages to 0 on azumuthal integration $d\phi$

change variables: $\sin\theta = \frac{a}{((Z-z)^2+a^2)^{1/2}}$, $\cos\theta = \frac{Z-z}{((Z-z)^2+a^2)^{1/2}}$,

$$\cos\theta\, d\theta = \frac{-a\,(Z-z)\,dZ}{((Z-z)^2+a^2)^{3/2}},\quad \frac{-dZ}{((z-Z)^2+a^2)^{3/2}} = \frac{\cos\theta\, d\theta}{a\,(Z-z)} = \frac{\sin\theta\, d\theta}{a^2}$$

$$\mathbf{B}(0,z) = -\hat{\mathbf{z}}\,\frac{\mu_0 M}{2}\,a^2\int_{-s/2}^{s/2}\frac{dZ}{((z-Z)^2+a^2)^{3/2}} = -\hat{\mathbf{z}}\,\frac{\mu_0 M}{2}\int_{\theta_1}^{\theta_2} d\theta\,\sin\theta$$

$$= \hat{\mathbf{z}}\,\frac{\mu_0 M}{2}(\cos\theta_1 - \cos\theta_2) = \hat{\mathbf{z}}\,\frac{\mu_0 M}{2}\left(\frac{-(z+s/2)}{((z+s/2)^2+a^2)^{1/2}} - \frac{-(z-s/2)}{((z-s/2)^2+a^2)^{1/2}}\right)$$