PH 431 Exam Equation Sheet Fall 2012

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc} \qquad \oint_S \vec{D}\cdot d\vec{a} = Q_{free,enc} \qquad \nabla\bullet\vec{E} = \frac{1}{\varepsilon_0}\rho \qquad \nabla\bullet\vec{D} = \rho_f$$

$$\oint_C \vec{E}\cdot d\vec{l} = 0 \qquad \oint_C \vec{E}\cdot d\vec{l} = 0 \qquad \nabla\times\vec{E} = 0 \qquad \nabla\times\vec{E} = 0$$

$$\oint_S \vec{B}\cdot d\vec{a} = 0 \qquad \oint_S \vec{B}\cdot d\vec{a} = 0 \qquad \nabla\bullet\vec{B} = 0 \qquad \nabla\bullet\vec{B} = 0$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \qquad \oint_C \vec{H}\cdot d\vec{l} = I_{free,enc} \qquad \nabla\times\vec{B} = \mu_0\vec{J} \qquad \nabla\times\vec{H} = \vec{J}_f$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}-\hat{r}'}{|\vec{r}-\vec{r}'|^2}\rho(\vec{r}')\,d\tau' \qquad \vec{E} = -\nabla V$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{|\vec{r}-\vec{r}'|}\rho(\vec{r}')\,d\tau' \qquad \nabla^2 V = -\frac{1}{\varepsilon_0}\rho$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times(\hat{r}-\hat{r}')}{|\vec{r}-\vec{r}'|^2}\,d\ell' = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')\times(\hat{r}-\hat{r}')}{|\vec{r}-\vec{r}'|^2}\,da' = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times(\hat{r}-\hat{r}')}{|\vec{r}-\vec{r}'|^2}\,d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{|\vec{r}-\vec{r}'|}\,d\ell' = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')}{|\vec{r}-\vec{r}'|}\,da' = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\,d\tau' \qquad \vec{B} = \nabla\times\vec{A}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\vec{E} \qquad \sigma_b = \vec{P}\bullet\hat{n} \qquad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\sigma_b}{|\vec{r}-\vec{r}'|}\,da' + \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_b}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{P} = \varepsilon_0\chi_e\vec{E} \qquad \rho_b = -\nabla\bullet\vec{P}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} = \frac{1}{\mu}\vec{B} \qquad \vec{J}_b = \nabla\times\vec{M} \qquad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b(\vec{r}')}{|\vec{r}-\vec{r}'|}\,da' + \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(\vec{r}')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

$$\vec{K}_b = \vec{M}\times\hat{n}$$

$$\vec{M} = \chi_m\vec{H}$$

$$B^{\perp}_{above} - B^{\perp}_{below} = 0 \qquad E^{\perp}_{above} - E^{\perp}_{below} = \frac{1}{\varepsilon_0}\sigma$$

$$\vec{B}^{\parallel}_{above} - \vec{B}^{\parallel}_{below} = \mu_0\left(\vec{K}\times\hat{n}\right) \qquad \vec{E}^{\parallel}_{above} - \vec{E}^{\parallel}_{below} = 0$$

$$H^{\perp}_{above} - H^{\perp}_{below} = -\left(M^{\perp}_{above} - M^{\perp}_{below}\right) \qquad D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$$

$$\vec{H}^{\parallel}_{above} - \vec{H}^{\parallel}_{below} = \vec{K}_f\times\hat{n} \qquad \vec{D}^{\parallel}_{above} - \vec{D}^{\parallel}_{below} = \vec{P}^{\parallel}_{above} - \vec{P}^{\parallel}_{below}$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + B_l r^{-(l+1)}\right)P_l(\cos\theta)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r}')\,d\tau'$$

$$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \left(3x^2-1\right)/2,\ P_3(x) = \left(5x^3-3x\right)/2$$

$$V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\bullet\hat{r}}{r^2} \qquad \vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta}\right)$$

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2} \qquad \vec{B}_{dip}(r,\theta) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta}\right)$$

$$\nabla \bullet \vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$U = \frac{1}{2}\int \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau$$

$$\vec{P} = \varepsilon_0 \int \left(\vec{E} \times \vec{B} \right) d\tau$$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B} \right)$$

$$P = \frac{\mu_0}{6\pi c} q^2 a^2$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k} \bullet \vec{r} - \omega t\right)}$$

$$\vec{B} = \frac{1}{c} \hat{k} \times \vec{E}$$

$$\vec{E} = \vec{E}_0\left(x, y\right) e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0\left(x, y\right) e^{i(kz - \omega t)}$$

$$\omega_{mn} = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$E_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega \frac{\partial B_z}{\partial y} \right)$$

$$E_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x} \right)$$

$$B_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2} \frac{\partial E_z}{\partial y} \right)$$

$$B_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2} \frac{\partial E_z}{\partial x} \right)$$

CARTESIAN COORDINATES

$$\nabla t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial y}\hat{y} + \frac{\partial t}{\partial z}\hat{z}$$

$$\hat{x} = \sin\theta\cos\phi\ \hat{r} + \cos\theta\cos\phi\ \hat{\theta} - \sin\phi\ \hat{\phi}$$
$$\hat{y} = \sin\theta\sin\phi\ \hat{r} + \cos\theta\sin\phi\ \hat{\theta} + \cos\phi\ \hat{\phi}$$
$$\hat{z} = \cos\theta\ \hat{r} - \sin\theta\ \hat{\theta}$$

$$\nabla \bullet \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \vec{v} = \left[\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right]\hat{x} + \left[\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right]\hat{y} + \left[\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right]\hat{z}$$

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

$$d\tau = dx\ dy\ dz$$

CYLINDRICAL COORDINATES

$$\nabla t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \varphi}\hat{\varphi} + \frac{\partial t}{\partial z}\hat{z}$$

$$\hat{s} = \cos\phi\ \hat{x} + \sin\phi\ \hat{y}$$
$$\hat{\phi} = -\sin\phi\ \hat{x} + \cos\phi\ \hat{y}$$
$$\hat{z} = \hat{z}$$

$$\nabla \bullet \vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(s\ v_s) + \frac{1}{s}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \vec{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\varphi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s\ v_\varphi) - \frac{\partial v_s}{\partial \varphi}\right]\hat{z}$$

$$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2}$$

$$d\tau = s\ ds\ d\varphi\ dz$$

SPHERICAL COORDINATES

$$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \varphi}\hat{\varphi}$$

$$\hat{r} = \sin\theta\cos\phi\ \hat{x} + \sin\theta\sin\phi\ \hat{y} + \cos\theta\ \hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\ \hat{x} + \cos\theta\sin\phi\ \hat{y} - \sin\theta\ \hat{z}$$
$$\hat{\phi} = -\sin\phi\ \hat{x} + \cos\phi\ \hat{y}$$

$$\nabla \bullet \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$$

$$\nabla \times \vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(v_\varphi \sin\theta) - \frac{\partial v_\theta}{\partial \varphi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r}(r \cdot v_\varphi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r \cdot v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\varphi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \varphi^2}$$

$$d\tau = r^2\sin\theta\ dr\ d\theta\ d\varphi$$

$$\int_0^{2\pi}(\sin nx)(\sin mx)\,dx = \pi\delta_{n,m}\ ,\ n,m \geq 1$$

$$\int_0^{2\pi}(\cos nx)(\cos mx)\,dx = \pi\delta_{n,m}\ ,\ n,m \geq 0$$