

PH 431 Exam Equation Sheet

Fall 2012

$$\begin{aligned}
 \oint_S \vec{E} \cdot d\vec{a} &= \frac{1}{\epsilon_0} Q_{enc} & \oint_S \vec{D} \cdot d\vec{a} &= Q_{free,enc} & \nabla \cdot \vec{E} &= \frac{1}{\epsilon_0} \rho & \nabla \cdot \vec{D} &= \rho_f \\
 \oint_C \vec{E} \cdot d\vec{l} &= 0 & \oint_C \vec{E} \cdot d\vec{l} &= 0 & \nabla \times \vec{E} &= 0 & \nabla \times \vec{E} &= 0 \\
 \oint_S \vec{B} \cdot d\vec{a} &= 0 & \oint_S \vec{B} \cdot d\vec{a} &= 0 & \nabla \cdot \vec{B} &= 0 & \nabla \cdot \vec{B} &= 0 \\
 \oint_C \vec{B} \cdot d\vec{l} &= \mu_0 I_{enc} & \oint_C \vec{H} \cdot d\vec{l} &= I_{free,enc} & \nabla \times \vec{B} &= \mu_0 \vec{J} & \nabla \times \vec{H} &= \vec{J}_f
 \end{aligned}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{r} - \hat{r}'}{|\vec{r} - \vec{r}'|^2} \rho(\vec{r}') d\tau' \quad \vec{E} = -\nabla V$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{|\vec{r} - \vec{r}'|} \rho(\vec{r}') d\tau' \quad \nabla^2 V = -\frac{1}{\epsilon_0} \rho$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times (\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} d\ell' = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}') \times (\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} da' = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{|\vec{r} - \vec{r}'|} d\ell' = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}')}{|\vec{r} - \vec{r}'|} da' = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \quad \vec{B} = \nabla \times \vec{A}$$

$$\begin{aligned}
 \vec{D} &= \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} & \sigma_b &= \vec{P} \cdot \hat{n} & V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b}{|\vec{r} - \vec{r}'|} da' + \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_b}{|\vec{r} - \vec{r}'|} d\tau' \\
 \vec{P} &= \epsilon_0 \chi_e \vec{E} & \rho_b &= -\nabla \cdot \vec{P}
 \end{aligned}$$

$$\begin{aligned}
 \vec{H} &= \frac{1}{\mu_0} \vec{B} - \vec{M} = \frac{1}{\mu} \vec{B} & \vec{J}_b &= \nabla \times \vec{M} & \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \oint_S \frac{\vec{K}_b(\vec{r}')}{|\vec{r} - \vec{r}'|} da' + \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}_b(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \\
 \vec{M} &= \chi_m \vec{H} & \vec{K}_b &= \vec{M} \times \hat{n}
 \end{aligned}$$

$$\begin{aligned}
 B_{above}^\perp - B_{below}^\perp &= 0 & E_{above}^\perp - E_{below}^\perp &= \frac{1}{\epsilon_0} \sigma \\
 \vec{B}_{above}^\parallel - \vec{B}_{below}^\parallel &= \mu_0 (\vec{K} \times \hat{n}) & \vec{E}_{above}^\parallel - \vec{E}_{below}^\parallel &= 0 \\
 H_{above}^\perp - H_{below}^\perp &= -(M_{above}^\perp - M_{below}^\perp) & D_{above}^\perp - D_{below}^\perp &= \sigma_f \\
 \vec{H}_{above}^\parallel - \vec{H}_{below}^\parallel &= \vec{K}_f \times \hat{n} & \vec{D}_{above}^\parallel - \vec{D}_{below}^\parallel &= \vec{P}_{above}^\parallel - \vec{P}_{below}^\parallel
 \end{aligned}$$

$$V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-(l+1)}) P_l(\cos \theta)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') d\tau'$$

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2, \quad P_3(x) = (5x^3 - 3x)/2$$

$$V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2} \quad \vec{E}_{dip}(r, \theta) = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$$

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \vec{B}_{dip}(r, \theta) = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$$

CARTESIAN COORDINATES

$$\nabla t = \frac{\partial t}{\partial x} \hat{x} + \frac{\partial t}{\partial y} \hat{y} + \frac{\partial t}{\partial z} \hat{z}$$

$$\hat{x} = \sin \theta \cos \phi \hat{r} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi}$$

$$\hat{y} = \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$$

$$\hat{z} = \cos \theta \hat{r} - \sin \theta \hat{\theta}$$

$$\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \vec{v} = \left[\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right] \hat{x} + \left[\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right] \hat{y} + \left[\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right] \hat{z}$$

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

$$d\tau = dx \, dy \, dz$$

CYLINDRICAL COORDINATES

$$\nabla t = \frac{\partial t}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{z}$$

$$\hat{s} = \cos \phi \hat{x} + \sin \phi \hat{y}$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$$

$$\hat{z} = \hat{z}$$

$$\nabla \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \vec{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{z}$$

$$\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

$$d\tau = s \, ds \, d\phi \, dz$$

SPHERICAL COORDINATES

$$\nabla t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$

$$\hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$$

$$\hat{\theta} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z}$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\nabla \times \vec{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (v_\phi \sin \theta) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r \cdot v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r \cdot v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

$$d\tau = r^2 \sin \theta \, dr \, d\theta \, d\phi$$

$$\int_0^{2\pi} (\sin nx)(\sin mx) \, dx = \pi \delta_{n,m}, \quad n, m \geq 1$$

$$\int_0^{2\pi} (\cos nx)(\cos mx) \, dx = \pi \delta_{n,m}, \quad n, m \geq 0$$