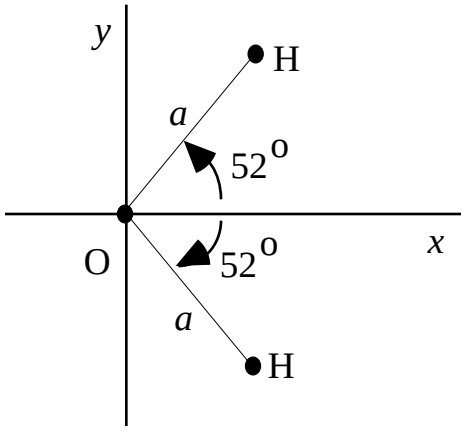
SOLUTION TO EXERCISE 1

A model of the water molecule H_2O is shown in MT Figure 9-B.



a. Where is the center of mass? X = ? Y = ? Z = ? let $\theta = 52^o$

$$m_O = 16 \text{ amu}, \; m_H = 1 \text{ amu}$$

$$x_H = a\cos\theta\,,\; x_O = 0,\; X = \frac{m_H x_H + m_H x_H + m_O x_O}{2m_H + m_O} = \frac{2}{18}\, a\cos\theta$$

$$y_H = \pm a\sin\theta\,,\; y_O = 0,\; Y = \frac{m_H y_H - m_H y_H + m_O y_O}{2m_H + m_O} = 0$$

$z_H = 0,\; z_O = 0,\; Z = 0$. Z and Y can be seen directly from symmetry

b. What is the moment of inertia about the x axis? I_x = ?

$$I_x = 2m_H y_H^2 + 2m_H z_H^2 + m_O y_O^2 + m_O z_O^2 = 2 \text{ amu } a^2\sin^2\theta$$

c. What is the moment of inertia about the y axis? I_y = ?

$$I_x = 2m_H x_H^2 + 2m_H z_H^2 + m_O x_O^2 + m_O z_O^2 = 2 \text{ amu } a^2\cos^2\theta$$

d. What is the moment of inertia about the z axis? I_z = ?

$$I_x = 2m_H x_H^2 + 2m_H y_H^2 + m_O x_O^2 + m_O y_O^2 = 2 \text{ amu } a^2(\sin^2\theta + \cos^2\theta) = 2 \text{ amu } a^2$$

e. What is the moment of inertia about an axis that goes through the hydrogen atoms? I' = ? $x' = x - x_H$, $y' = y$, $z' = z$

$$I' = m_O x'^2_O + 0 + 0 + 0 + 0 + 0 = (8 \text{ amu})\,(a\cos\theta)^2$$