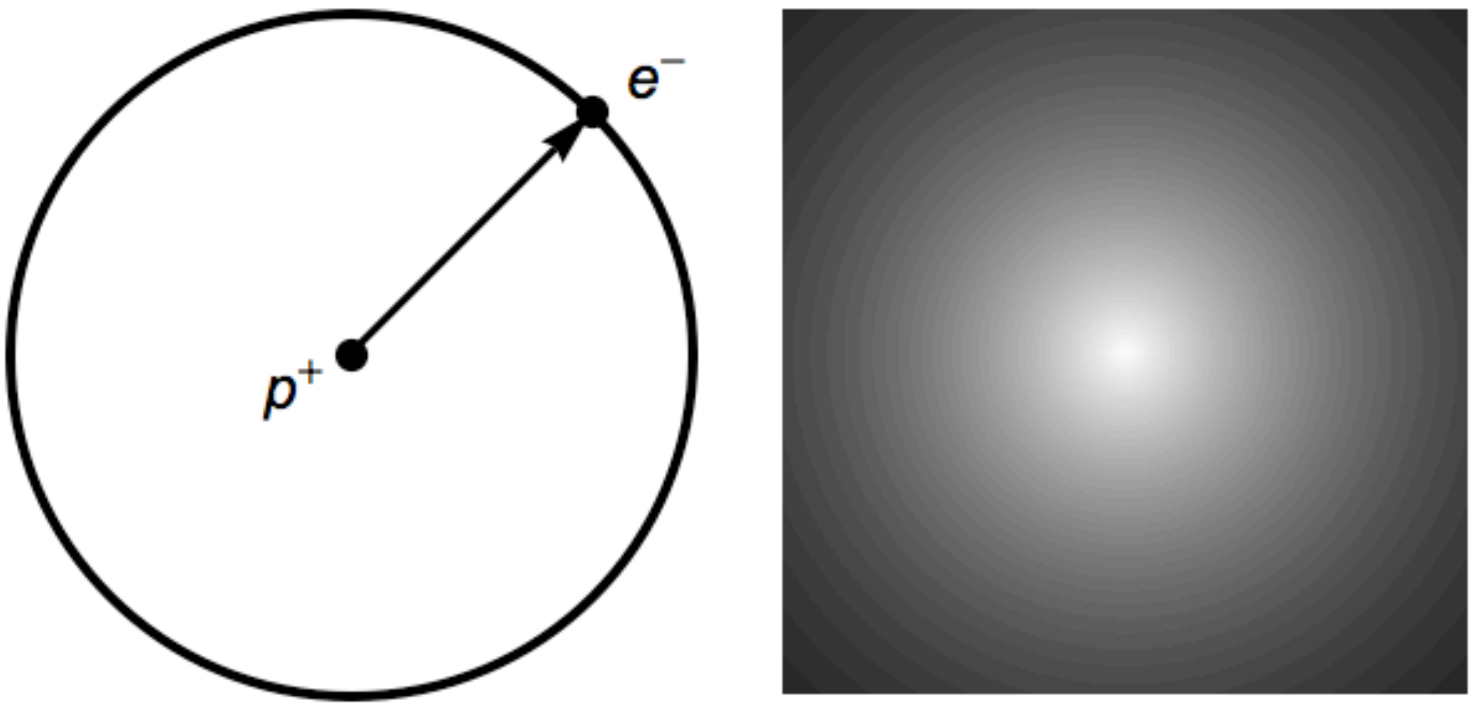
# L1 – Center of mass, reduced mass, angular momentum

Read course notes sections 2-6

Taylor 3.3-3.5

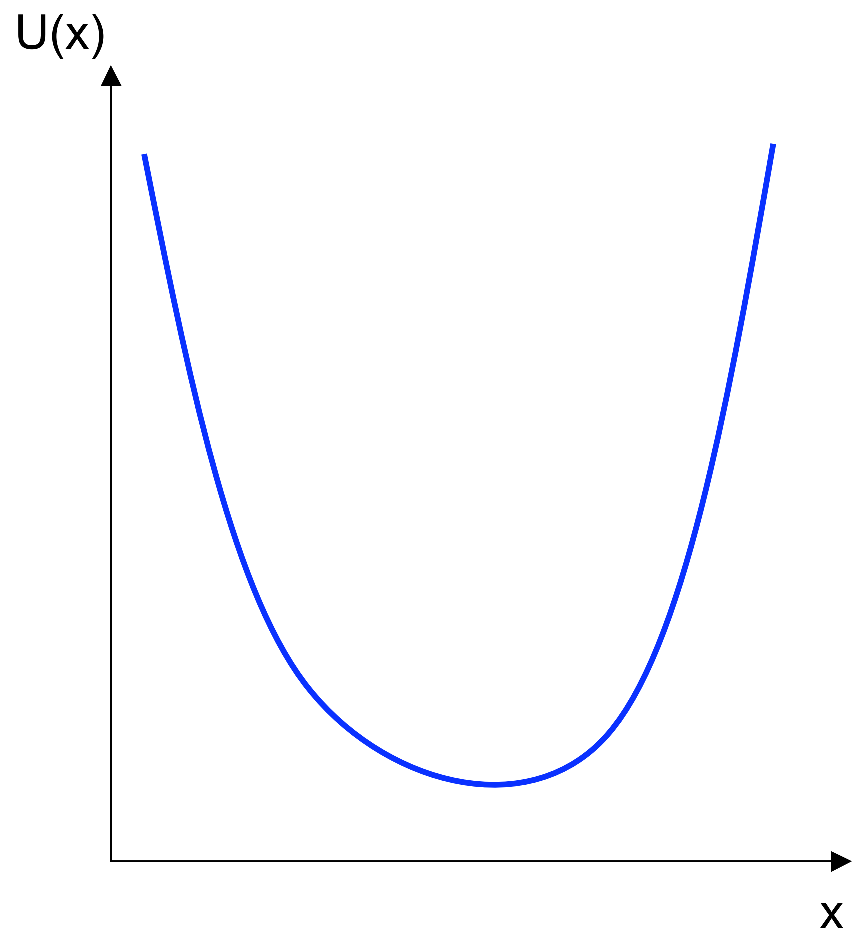
# Planets and atoms
# Classical and Quantum




(a) (b)

# Review: Forces, Potential Energy

| Force | Potential |
|---|---|
| $mg\hat{\mathbf{j}}$ | $mgh$ |
| $-kx\hat{\mathbf{i}}$ | $\frac{1}{2}kx^2$ |
| $-\frac{GMm}{r^2}\hat{\mathbf{r}}$ | $-\frac{GMm}{r}$ |
| $\mathbf{F}$ | $\Delta U = -\int_i^f \mathbf{F} \bullet d\mathbf{r}$ |



# Review: Newton, Energy solutions

Newton's 2nd Law

$$F = ma$$

$$\ddot{x} = \frac{F}{m}$$

$$\iint$$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Consevation of Energy

$$E = T + U$$

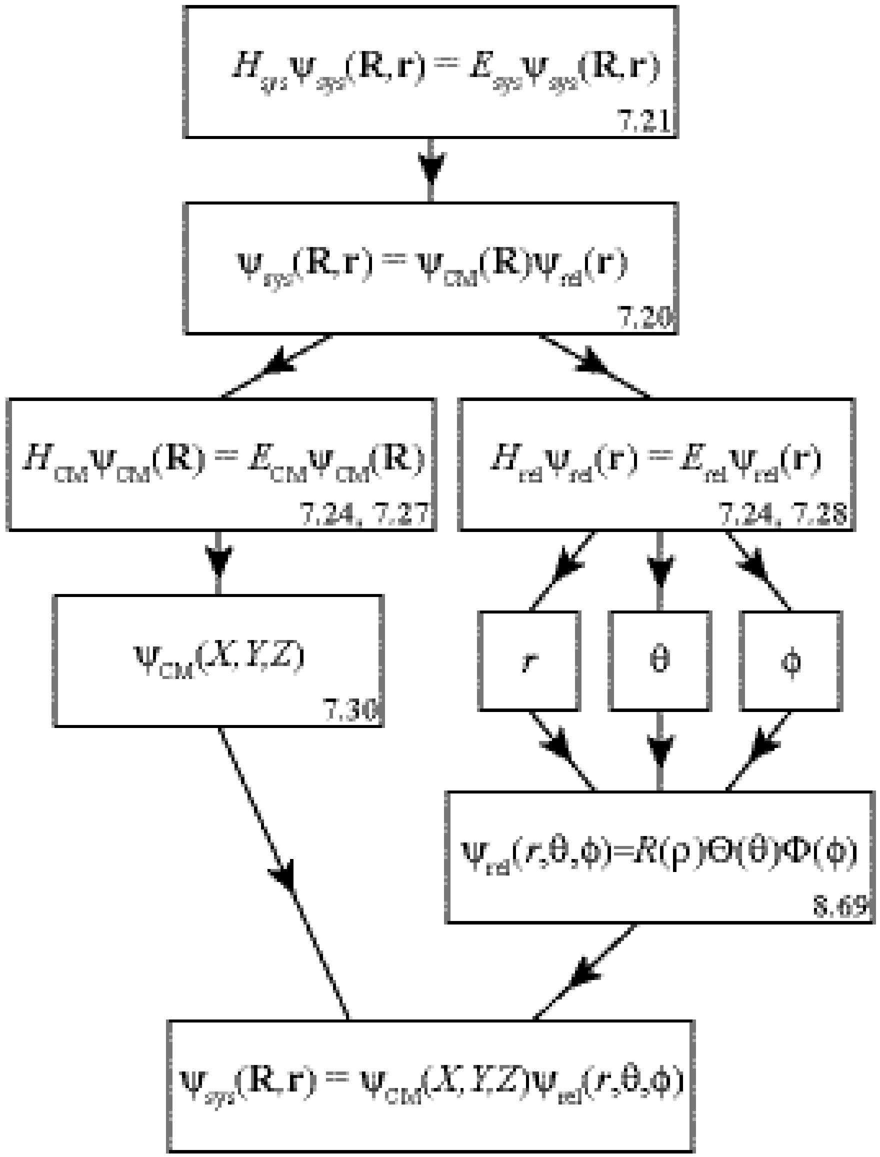
$$E = \tfrac{1}{2} m \dot{x}^2 + U(x)$$

$$\dot{x} = \pm \sqrt{\frac{2\left[E - U(x)\right]}{m}}$$

$$t = \int \frac{dx}{\sqrt{\frac{2\left[E - U(x)\right]}{m}}}$$

$$t(x) \quad \Rightarrow \quad x(t)$$

# 2 bodies, 3 dimensions, ouch!

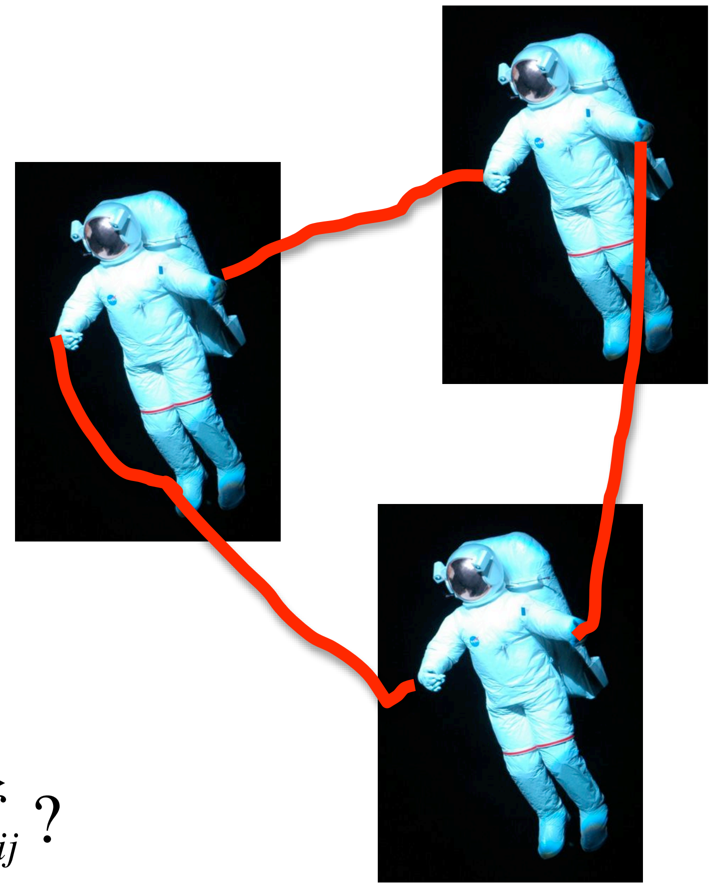


# Newton's Law for 3 bodies (or *n*)

$$m_1 \frac{d^2 \vec{r}_1}{dt^2} = \vec{F}_1 + 0 + \vec{f}_{12} + \vec{f}_{13}$$

$$m_2 \frac{d^2 \vec{r}_2}{dt^2} = \vec{F}_2 + \vec{f}_{21} + 0 + \vec{f}_{23}$$

$$m_3 \frac{d^2 \vec{r}_3}{dt^2} = \vec{F}_3 + \vec{f}_{31} + \vec{f}_{32} + 0$$



What are $m_i$ ? $\vec{r}_i$ ? $\vec{F}_i$ ? $\vec{f}_{ij}$ ?

Coupled Equations: Hard!! Use Newton's 3rd Law

# Decouple center of mass motion

- Position of center of mass is the (mass) weighted average of individual mass positions

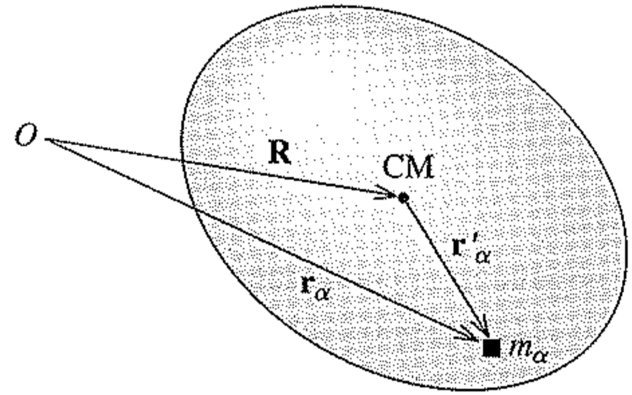
$$\vec{R}_{CM} \equiv \frac{m_1}{M}\vec{r}_1 + \frac{m_2}{M}\vec{r}_2 + \frac{m_3}{M}\vec{r}_3 + \ldots = \sum_n \frac{m_n}{M}\vec{r}_n$$

$$\sum_n m_n \frac{d^2\vec{r}_n}{dt^2} = \sum_n \vec{F}_n$$

$$\frac{d^2}{dt^2}\sum_n m_n\vec{r}_n = \sum_n \vec{F}_n \quad \Rightarrow \quad M\frac{d^2\vec{R}_{CM}}{dt^2} = \sum_n \vec{F}_n = \vec{F}_{ext}$$

$$M\frac{d^2\vec{R}_{CM}}{dt^2} = 0 \Rightarrow ?$$

# CM importance




$$\vec{L}_{TOT} = \sum_i \vec{r}_i \times \vec{p}_i = \sum_i \vec{r}_i \times m_i \dot{\vec{r}}_i = \sum_i \left(\vec{R} + \vec{r}_i'\right) \times m_i \left(\dot{\vec{R}} + \dot{\vec{r}}_i'\right)$$

$$= \vec{R} \times \vec{P} + \sum_i \vec{r}_i' \times m_i \dot{\vec{r}}_i' = \boxed{\vec{L}_{of\ CM} + \vec{L}_{about\ CM}}$$

$$T_{TOT} = \sum_i \tfrac{1}{2} m_i \dot{\vec{r}}_i^{\,2} = \sum_i \tfrac{1}{2} m_i \left(\dot{\vec{R}} + \dot{\vec{r}}_i'\right)^2$$

$$= \tfrac{1}{2} M \dot{\vec{R}}^2 + \tfrac{1}{2} \sum_i m_i \dot{\vec{r}}'^{\,2} = \boxed{T_{of\ CM} + T_{about\ CM}}$$