Quantum Calculations on a Ring I

Before you begin, recall that an arbitrary state $|\Phi\rangle$ can be written as

$$|\Phi\rangle \doteq \begin{pmatrix} \vdots \\ \langle 4|\Phi\rangle \\ \langle 3|\Phi\rangle \\ \langle 2|\Phi\rangle \\ \langle 1|\Phi\rangle \\ \langle 0|\Phi\rangle \\ \langle -1|\Phi\rangle \\ \langle -2|\Phi\rangle \\ \langle -3|\Phi\rangle \\ \langle -4|\Phi\rangle \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \\ a_{-1} \\ a_{-2} \\ a_{-3} \\ a_{-4} \\ \vdots \end{pmatrix}$$

In this activity, your group will carry out calculations on each of the following normalized abstract quantum states on a ring:

$$|\Phi_a\rangle = \sqrt{\frac{2}{12}}|3\rangle + \sqrt{\frac{1}{12}}|2\rangle + \sqrt{\frac{3}{12}}|0\rangle + \sqrt{\frac{2}{12}}|-1\rangle + \sqrt{\frac{1}{12}}|-3\rangle + \sqrt{\frac{3}{12}}|-4\rangle$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} 0 \\ \sqrt{\frac{2}{12}} \\ \sqrt{\frac{1}{12}} \\ 0 \\ \sqrt{\frac{3}{12}} \\ \sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}} \end{pmatrix}$$

$$\Phi_c(\phi) = \sqrt{\frac{1}{24\pi}}\left(\sqrt{2}e^{i3\phi} + e^{i2\phi} + \sqrt{3} + \sqrt{2}e^{-i1\phi} + e^{-i3\phi} + \sqrt{3}e^{-i4\phi}\right)$$

For each of the following 5 questions, state the postulate of quantum mechanics you use to complete the calculation and show explicitly how you use that postulate to answer the question.

1. If you measured the z-component of angular momentum for each state, what is the probability that you would obtain:
   Group 1: $2\hbar$ Group 2: $-4\hbar$ Group 3: $-3\hbar$ Group 4: $4\hbar$
   Group 5: $-1\hbar$ Group 6: 0 Group 7: $-2\hbar$ Group 8: $3\hbar$

2. If you measured the energy for each state, what is the probability that you would obtain:
   Group 1: $\frac{9\hbar^2}{2I}$ Group 2: 0 Group 3: $\frac{4\hbar^2}{2I}$ Group 4: $\frac{1\hbar^2}{2I}$
   Group 5: $\frac{16\hbar^2}{2I}$ Group 6: $\frac{1\hbar^2}{2I}$ Group 7: $\frac{9\hbar^2}{2I}$ Group 8: $\frac{4\hbar^2}{2I}$

3. How are the calculations you made for the different state representations similar and different from each other? Be prepared to compare and contrast the calculations you made for each of the different representations (ket, matrix, wavefunction).

4. If you measured the z-component of angular momentum, what possible values could you have obtained with non-zero probability?

5. If you measured the energy, what possible values could you have obtained with non-zero probability?

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