Central Forces Homework 4

Due 03/09/12

PRACTICE:

1. (McIntyre 7.5)

   An angular momentum system with $l = 1$ is prepared in the state:

   $$|\psi\rangle = \frac{2}{\sqrt{29}}|1,1\rangle + i\frac{3}{\sqrt{29}}|1,0\rangle - \frac{4}{\sqrt{29}}|1,-1\rangle$$

   (a) What are the possible results of a measurement of the angular momentum component L_z, and with what probabilities would they occur?

   (b) What are the possible results of a measurement of the angular momentum component L_x, and with what probabilities would they occur?

   (c) Plot histograms of the predicted measurement results from parts (a) and (b).

REQUIRED:

2. (McIntyre 7.10) Use the separation of variables procedure in Appendix E on the angular equation (Eq. 7.80 on p. 217) to obtain Eq. 7.82 and Eq. 7.83 for the polar and azimuthal angles.

3. (McIntyre 7.13)

   Consider the normalized state $|\Phi\rangle$ for a quantum mechanical particle of mass μ constrained to move on a circle of radius r_0, given by:

   $$|\Phi\rangle = \tfrac{\sqrt{3}}{2}\,|3\rangle + \tfrac{i}{2}\,|-2\rangle$$

   (a) What is the probability that a measurement of L_z will yield $2\hbar$? $3\hbar$?

   (b) What is the probability that a measurement of energy will yield $E = \frac{2\hbar^2}{I}$?

   (c) What is the expectation value of L_z in this state?

   (d) What is the expectation value of the energy in this state?

4. (McIntyre 7.15)

   Consider the normalized wavefunction $\Phi(\phi)$ for a quantum mechanical particle of mass μ constrained to move on a circle of radius r_0, given by:

$$\Phi(\phi) = \frac{N}{2 + \cos(3\phi)}$$

   where N is the normalization constant.

   (a) Find N.
   (b) Plot this wave function.
   (c) What is the expectation value of L_z in this state?

5. Attached, you will find a table showing different representations of physical quantities associated with a particle-in-a-box. Make a similar table for a particle confined to a ring. Include all of the following information.

   - Hamiltonian
   - Eigenvalues of Hamiltonian
   - Normalized eigenstates of Hamiltonian
   - Coefficient of the nth eigenstate
   - Probability of measuring E_n
   - Expectation value of Hamiltonian
   - Z-component of angular momentum
   - Eigenvalues of z-component of angular momentum
   - Eigenstates of z-component of angular momentum
   - Coefficient of mth state of z-component of angular momentum
   - Probability of measuring $m\hbar$ for z-component of angular momentum
   - Expectation value of z-component of angular momentum

Particle in a Box

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian	$\hat{H}$	$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$	$\begin{pmatrix} E_1 & 0 & 0 & \cdots \\ 0 & E_2 & 0 & \cdots \\ 0 & 0 & E_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of Hamiltonian	$E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2$	$E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2$	$E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2$
Normalized Eigenstates of Hamiltonian	$\lvert n\rangle$	$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \cdots$
Coefficient of n^{th} energy eigenstate	$c_n = \langle n \vert \psi\rangle$	$c_n = \int_0^L \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\psi(x)\,dx$	$\begin{pmatrix} 0 & \cdots & 1 & \cdots \end{pmatrix}\begin{pmatrix} c_1 \\ \vdots \\ c_n \\ \vdots \end{pmatrix}$
Probability of measuring E_n	$P(E_n) = \lvert c_n\rvert^2 = \lvert\langle n \vert \psi\rangle\rvert^2$	$P(E_n) = \lvert c_n\rvert^2 = \left\lvert\int_0^L \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\psi(x)\,dx\right\rvert^2$	$P(E_n) = \lvert c_n\rvert^2 = \left\lvert\begin{pmatrix} 0 & \cdots & 1 & \cdots \end{pmatrix}\begin{pmatrix} c_1 \\ \vdots \\ c_n \\ \vdots \end{pmatrix}\right\rvert^2$
Expectation value of Hamiltonian	$\langle\psi\vert H\vert\psi\rangle = \sum_n \lvert c_n\rvert^2 E_n$	$\langle\psi\vert H\vert\psi\rangle = \int_0^L \psi^*(x)\hat{H}\psi(x)\,dx$	$\langle\psi\vert H\vert\psi\rangle = \begin{pmatrix} c_1^* & c_2^* & \cdots \end{pmatrix}\begin{pmatrix} E_1 & 0 & \cdots \\ 0 & E_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$

Particle on a Ring

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian			
Eigenvalues of Hamiltonian			
Normalized Eigenstates of Hamiltonian			
Coefficient of m^{th} energy eigenstates			
Probability of measuring E_m			
Expectation value of Hamiltonian			

Particle on a Ring

Operator for z-component of angular momentum			
Eigenvalues of z-component of angular momentum			
Normalized Eigenstates of z-component of angular momentum			
Coefficient of m^{th} eigenstates of z-component of angular momentum			
Probability of measuring $m\hbar$ for z-component of angular momentum			
Expectation value of z-component of angular momentum			