Central Forces Homework 1

Due 2/29/12

PRACTICE:

1. If a central force is the only force acting on a system of two masses (i.e. no external forces), what will the motion of the center of mass be?

2. Which of the following forces can be central forces? which cannot?

   (a) The force on a test mass m in a gravitational field $\vec{g}$, i.e. $m\vec{g}$

   (b) The force on a test charge q in an electric field $\vec{E}$, i.e. $q\vec{E}$

   (c) The force on a test charge q moving at velocity $\vec{v}$ in a magnetic field $\vec{B}$, i.e. $q\vec{v}\times\vec{B}$

3. Show that the plane polar coordinates we have chosen are equivalent to spherical coordinates if we make the choices:

   (a) The direction of z in spherical coordinates is the same as the direction of $\vec{L}$.

   (b) The θ of spherical coordinates is chosen to be $\pi/2$, so that the orbit is in the equatorial plane of spherical coordinates.

4. Show that the plane of the orbit is perpendicular to the angular momentum vector $\vec{L}$.

5. Using your favorite graphing package, make a plot of the reduced mass μ as a function of m_1 and m_2. What about the shape of this graph tells you something about the physical world that you would like to remember. You should be able to find at least three things.

REQUIRED:

6. Consider a system of two particles.

   (a) Show that the total kinetic energy of the system is the same as that of two "fictitious" particles: one of mass $M = m_1 + m_2$ moving with the speed of the CM (center of mass) and one of mass μ (the reduced mass) moving with the speed of the relative position $\vec{r} = \vec{r}_2 - \vec{r}_1$.

   (b) Show that the total angular momentum of the system can be similarly decomposed into the angular momenta of these two fictitious particles.

7. Consider two particles of equal mass m. The forces on the particles are $\vec{F}_1 = 0$ and $\vec{F}_2 = F_0\hat{\imath}$. If the particles are initially at rest at the origin, find the position, velocity, and acceleration of the center of mass as functions of time. Solve this problem in two ways, with or without theorems about the center of mass motion. Write a short description comparing the two solutions.

8. (a) Find $\mathbf{r}_{\text{sun}} - \mathbf{r}_{\text{cm}}$ and μ for the Sun–Earth system. Compare $\mathbf{r}_{\text{sun}} - \mathbf{r}_{\text{cm}}$ to the radius of the Sun and to the distance from the Sun to the Earth. Compare μ to the mass of the Sun and the mass of the Earth.

   (b) Repeat the calculation for the Sun–Jupiter system.