Homework 3

Due Wednesday 12/4 in class

1) Calculate by hand using paper and pencil (no computers) the Fourier transform of the LRC impulse response function $\chi(t)$:

$$\chi(t) = \frac{1}{L} e^{-\beta t}\left[\cos\omega_1 t - \frac{\beta}{\omega_1}\sin\omega_1 t\right]\theta(t),$$

and show that it is equal to the admittance (1/Z(ω)) (modulo a factor of $\sqrt{2\pi}$).

2) Show that, in the case of very small damping ($\beta << \omega_0$) and frequencies close to resonance, the real part of the complex admittance can be written simply as

$$\mathrm{Re}\left(\frac{1}{Z(\omega)}\right) = \frac{1}{R} L(\omega),$$

where $L(\omega)$ is a Lorentzian function that has the form

$$L(\omega) = \frac{\left(\frac{\Delta\omega}{2}\right)^2}{(\omega - \omega_0)^2 + \left(\frac{\Delta\omega}{2}\right)^2}.$$

The Lorentzian is centered at ω_0 and has a full-width-half-maximum (FWHM) of $\Delta\omega$. Determine the width $\Delta\omega$ in terms of the defined circuit parameters. Plot this new approximate expression with the exact expression for comparison.

3) Show that, given the same conditions as in (2) above, the quality factor can be expressed as

$$Q = \frac{\omega_0}{\Delta\omega}.$$