Capacitors and Time-Dependent Signals

Concept

The purpose is to learn about time-dependent (AC) analysis of RC circuits using a function generator and an oscilloscope. The transient response of an RC circuit will be studied in the time-domain using first a switch and a DMM and then the combination of a square-wave from a function generator and an oscilloscope. Frequency-domain behavior will be measured as well, and the response function of RC circuits will be determined. Complex impedance will be introduced, and Fourier analysis of complicated waveforms will be presented.

Helpful hints and warnings

The "ground symbol" in a circuit implies that the grounds (outer conductors or shields) of the signal generator and the oscilloscope are connected to the circuit at this point. Unlike the DMM, the signal generator and oscilloscopes grounds can be connected only to the circuit ground. Thus, in the low-pass RC circuit, the oscilloscope can be used only to measure the potential across the capacitor. Conversely, in the high-pass CR circuit, the scope can be used only to measure the potential across the resistor.

To read the capacitance on the brown plastic capacitors, look for three numbers such as 153. The first two digits are the first two digits of the capacitance. The third digit is the order of magnitude or power of ten. So, 153 means a capacitance of $15 \times 10^3 = 15{,}000$ something. To figure out what "something" is, the type and size of the capacitor needs to be considered. In this case, the unit is picoFarad or pF. So, 15,000 pF = 15 nF = 0.015 μF. As with resistors, do not rely on the code for an accurate value of C. Always measure the capacitance using the *LRC meter*, which can also measure the inductance L.

Three types of capacitors are use in this laboratory. *Ceramic* capacitors exhibit a low capacitance/volume value, a high maximum potential difference rating and low *inductance*, which makes them suitable for high frequency applications. *Polymer* or *plastic* capacitors have a higher C/volume value, a lower maximum potential difference rating and a higher inductance, making them suitable for medium frequency applications. *Electrolytic* capacitors have a large C/volume value, a low working maximum potential difference and slow time response. Furthermore, electrolytics are *polarized*, that is, one side must always be positive with respect to the other to avoid electrochemistry, heating and component failure. As an example of the effect of improper orientation in a circuit, if you expect a potential of 10 V across the capacitor, you will measure only 8 V and power will be dissipated in the device.

Be sure to vary the frequency of the applied signal over a wide range, such as 100 Hz to 1 MHz, to make sure that you are working in the right range. You will need to measure the frequency ν each time you change it. Do so by measuring the *period* T as precisely as possible and using $\nu = 1/T$. The horizontal time scale on the oscilloscope is valid only when the *calibrate knob* is properly positioned. Remember the 2π! The *angular frequency* ω used in expressions is $\omega = 2\pi\nu$.

Since the *return* or ground line of the oscilloscope is connected to earth ground, it is possible to observe the time-dependent potential difference $V(t)$ only between a point in the circuit and earth ground. You cannot measure $V(t)$ across an individual resistor or capacitor unless one side is connected to earth ground.

Experimental Instructions

1. Time-dependent analysis of RC circuits

a. Experiment using manual switches for charge-discharge cycles.

(i) Build this circuit with $R < 100\ k\Omega$ and an RC product of about 0.01 s. For switches, you can simply use wires plugged into or out of your protoboard.

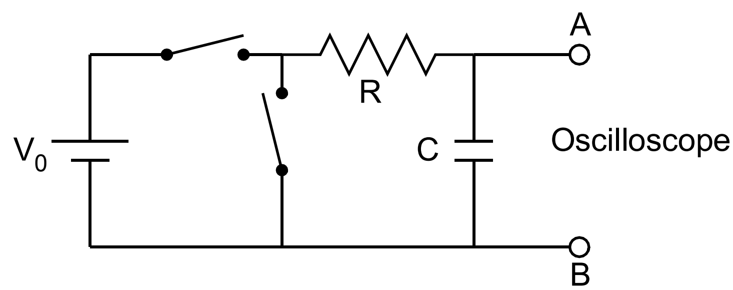

Figure 1: Manual switches to charge and discharge a capacitor.

(ii) To charge the capacitor, open the vertical switch to ground before closing the horizontal switch connecting the voltage source. To discharge the capacitor, open or *break* the connection to the power supply before *making* or closing the connection to ground.

(iii) The object is to determine the "$1/e$" rise and fall time (τ, a *characteristic time*) of the potential V_c across the capacitor for your particular RC combination. Ideally, $\tau = RC$ and $V_C(t) = V_0\, e^{-t/\tau}$ for discharging and $V_C(t) = V_0\left(1 - e^{-t/\tau}\right)$ for charging.

(iv) Capture a trace on the oscilloscope and use that to determine τ.

b. Experiment using a square wave signal from the function generator instead of mechanical switches.

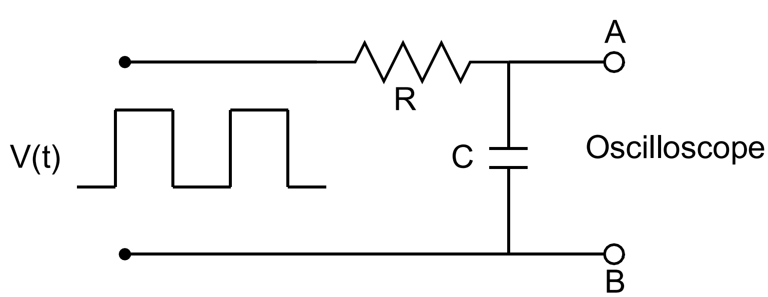

Figure 2: Use a unipolar (no negative values) square wave as input.

(i) Choose values of R and C such that $\tau = RC \approx 0.0001$ second.
(ii) Set the function generator to provide a 100 Hz square wave with no offset. Connect the output of the function generator to channel 1 of the oscilloscope and to the RC circuit. Connect the output of the circuit to channel 2. Connect the TTL output (a 5 V square wave) to the external trigger input of the scope. On the trigger menu set the trigger source to external and the trigger level to + 0.7 V. This will provide a reliable trigger regardless of the amplitude of the signals on channels 1 and 2. Set the oscilloscope to display one cycle and to utilize the full vertical range of the display.
(iii) Use the waveform acquisition program to acquire channels 1 and 2 simultaneously. The graphs that are automatically generated can be saved as pdf files. The program will ask for the name of a cvs file in which to save the waveforms. Excel or another plotting program can be used to read the cvs file and plot the two waveforms.
(iv) Measure τ by determining the time for the output to drop to $1/e$ of the maximum and to rise to $1-1/e$ of the maximum. Are these two values of τ equal? Does either equal the product RC? Explain any differences you observe.
(v) Vary the frequency from 0 to 100 kHz and determine the frequency range over which the behavior of the circuit is similar to the 100 Hz behavior.

2. The RC Integrator (Low-Pass Filter)

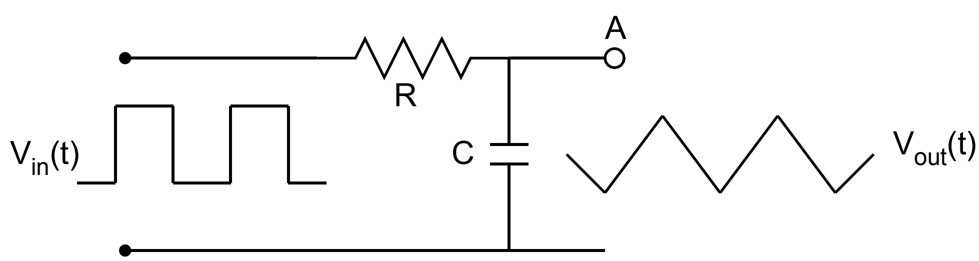

Figure 3: At a sufficiently high frequency, the RC circuit becomes an integrator.

a. Use the circuit of Part 1(b) and apply a 100 kHz square-wave signal.
b. Explain how the observed waveform is consistent with the concept of an RC circuit behaving as an integrator.
c. Over what frequency range does the circuit behave as an integrator, that is, capable of producing a triangle-wave output from a square-wave input? Explain why this circuit is also known as a low-pass filter.
d. Apply a triangle wave input at 100 kHz and explain the observed waveform.

3. The CR Differentiator (High-Pass Filter)

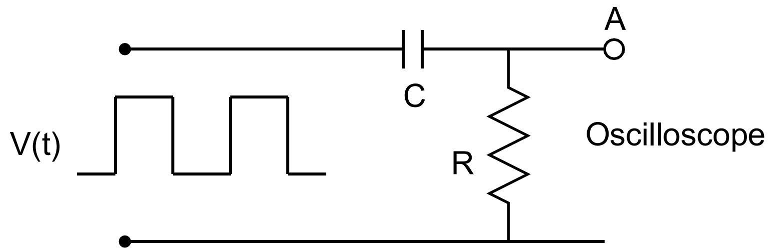

Figure 4: Use a unipolar (no negative values) square wave as input.

a. Reverse the positions of R and C, as shown in Fig. 4. Use the same values as in parts 1(b) and 2.

b. Vary the frequency from 0 to 100 kHz and describe the behavior of this circuit. Does it ever appear to behave as an integrator or a differentiator? Explain why this circuit is also known as a high-pass filter.

c. Apply a triangle-wave input at 100 Hz and explain the observed waveform.

4. Response of both filters to complex waveforms

a. The concept is to apply a visually complex waveform with three frequency components and observe the effects of transmission through the RC and CR configurations. Begin with $V_0(t) = \cos\omega_0 t$. Modulation of the signal amplitude with the function $A(t) = \frac{1}{2}(1+\cos\omega_m t)$ yields a new signal

$$V(t) = A(t)V_0(t) = \frac{1}{2}\cos\omega_0 t + \frac{1}{2}\cos\omega_m t\ \cos\omega_0 t$$

$$= \frac{1}{2}\cos\omega_0 t + \frac{1}{4}\left[\cos(\omega_0+\omega_m)t + \cos(\omega_0-\omega_m)t\right].$$

The Fourier transform of $V(t)$ yields a spectrum $F(\omega)$ with only three frequency components at ω_0 and $\omega_0 \pm \omega_m$. The power spectrum is proportional to $|F(\omega)|^2$, and, when represented as $20 \log|F(\omega)|$ in dB, it will exhibit peaks at $\omega_0 \pm \omega_m$ that are smaller than the peak at ω_0 by 6 dB.

b. Set the function generator to produce a sine wave at 10 kHz, with 10 V peak-to-peak amplitude and no offset. Push the modulation button and set the modulation to AM (amplitude modulation), 9 kHz and 100% depth of modulation. Observe the complex waveform on channel 1 versus time. Adjust the time scale so that you can observe two cycles of the lowest frequency modulation. Press the FFT button on either the on-screen menu or the math mode menu. Adjust the horizontal scale to clearly see all three frequency components. Verify that they are the expected frequencies with the expected amplitudes.

c. Apply the modulated signal to the RC configuration, and capture the input and output waveforms as functions of time. Use the FFT scope option to generate plots of the input and output power spectra. Save files with the csv extension so that graphs can be generated later. Interpret the appearance of the output waveform versus the input waveform in the time domain and the differences in the power spectra in the frequency domain.

d. Apply the modulated signal to the CR configuration, and perform the same analysis.

e. Draw a conclusion about the behavior or functionality of each configuration.

5. Frequency response of both low-pass and high-pass filters

a. The goal of this experiment is to measure the frequency dependence of each filter's response to a sine wave input. Measure both the amplitude and phase shift of the filter output relative to the input signal. Use the same R and C values that you used in the filter experiments above. Set the function generator to provide a sine wave with no offset and connect the output of the function generator to channel 1 of the oscilloscope and to the filter. Connect the output of the filter to channel 2. Use the TTL output to trigger the scope. Manually vary the frequency from 1 Hz to 1 MHz and observe the variation in amplitude and phase of the output relative to the input. Make a series of at least 20 measurements over the full frequency range. Because you will be plotting your data on a logarithmic plot, make at least two measurements per decade of frequency. Determine the transmission function $A(\nu)$ by dividing the output amplitude by the input amplitude. Be sure to measure the input amplitude from the function generator at each frequency, because the combination of your circuit and the limitations of the generator may lead to a signal that changes in amplitude with frequency. Measure the phase difference $\phi(\nu)$ between the output and input signals. A good point on the waveform to use for such measurements is the point at which the trace crosses 0 Volts (i.e. ground). If the period of the input signal is T and the displacement of the output signal zero-crossing from the input signal zero-crossing is t, then the phase difference is $\phi = 2\pi t / T$. If the output signal zero-crossing occurs after the input signal zero-crossing, then that is a phase lag or a negative phase.

c. For each filter, plot the data and theoretical curves together. For the amplitude plots, use the decibel (dB) scale $20 \log_{10} A(\nu)$ for the vertical axis and $\log \nu$ for the horizontal axis. For the phase plots, use a linear vertical axis for the phase and $\log \nu$ for the horizontal axis. Determine the *breakpoint* or *characteristic* frequency from the data plots by identifying the -3dB point on the amplitude plot (also known as a Bode plot) and the 45° point of the phase plot. Compare these to the expected theoretical value. Draw conclusions about the behavior of both circuits.

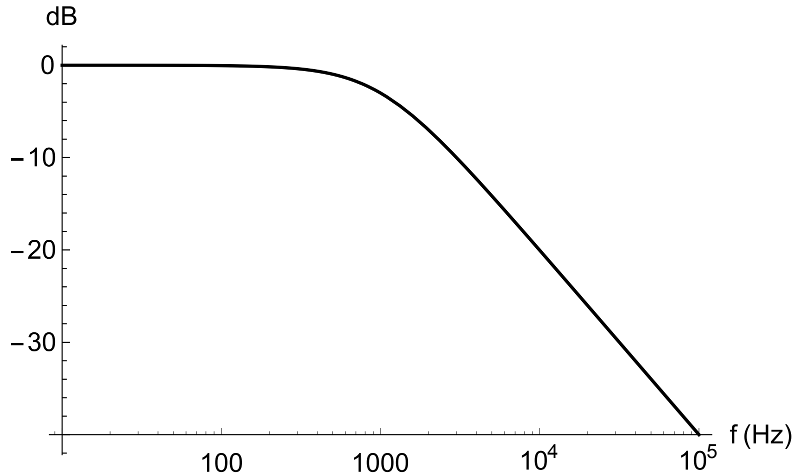

Figure 5: Bode plot for a low-pass RC filter.