The voltage divider circuit is shown in Fig. 1.

![Voltage divider circuit](image)

Figure 1: (a) Voltage divider circuit, and (b) defined currents.

We want to find the load voltage $V_{AB}$. Using the defined currents, we analyze the circuit with Kirchoff's laws for junctions and loops. The junction law yields

$$I_1 = I_2 + I_3 \quad (0.1)$$

The loop law applied to the two loops gives

$$V_S - I_1 R_1 - I_2 R_2 = 0 \quad (0.2)$$

$$I_2 R_2 - I_3 R_L = 0 \quad (0.3)$$

Note that we have 3 equations and 3 unknowns (3 currents). We want to find $V_{AB} = I_3 R_L$ so try to eliminate $I_1$ and $I_2$. Start with Eq. (0.3):

$$I_2 = I_3 \frac{R_L}{R_2} \quad (0.4)$$

Combine Eqs. (0.1), (0.2), and (0.4) to get

$$V_S = I_1 R_1 + I_2 R_2 = (I_2 + I_3) R_1 + I_2 R_2$$

$$= I_3 R_1 + (R_1 + R_2) I_3 \frac{R_L}{R_2} = I_3 \left( R_1 + (R_1 + R_2) \frac{R_L}{R_2} \right) \quad (0.5)$$

Solve for $I_3$ to get

$$I_3 = \frac{V_S}{R_1 + (R_1 + R_2) \frac{R_L}{R_2}} = \frac{V_S R_2}{R_1 R_2 + (R_1 + R_2) R_L} \quad (0.6)$$

The voltage across the load is

$$V_{AB} = I_3 R_L = \frac{R_2 R_L}{R_1 R_2 + (R_1 + R_2) R_L}$$

$$= \frac{V_S}{1 + \frac{R_1 (R_2 + R_L)}{R_2 R_L}} = \frac{V_S}{1 + \frac{1}{R_1 \left( \frac{1}{R_2} + \frac{1}{R_L} \right)}} \quad (0.7)$$
To make the plot, we assume the values $R_1 = 100\,\Omega$ and $R_2 = 100\,\Omega$ and $V_s = 5\,V$. The plot in Fig. (2) was made in Mathematica, with the load chosen over a range that shows the full asymptotic nature of the result. The important physics is that the voltage divider with $R_1 = R_2$ is expected to provide an output voltage of one-half of the input voltage, but it does so only in the limit that $R_L \gg R_{1,2}$.

![Figure 2: Output of voltage divider circuit.](image)