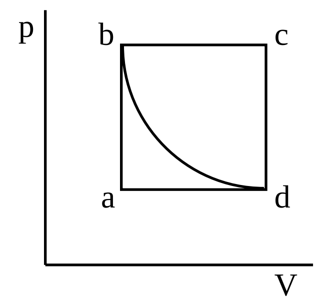
- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.
- Charges labeled +*q* are positive and those labeled -*q* are negative.

1. (25 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D) on the page for problem 1 in your exam booklet.

i) An ideal gas is taken through two closed cycle processes. Process A is the cycle *abda*, and process B is the cycle *bcdb* in the figure at right. For which process is the amount of heat transferred to the gas the largest?



A) Process A B) Process B C) Both the same.

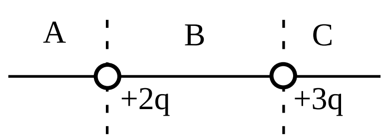
ii) An ideal gas undergoes an isobaric (constant pressure) compression. Does the temperature of the gas increase, decrease, or remain the same?

A) Increases. B) Decreases. C) Remains the same.

iii) The pressure on an ideal gas is increased at constant volume. Does the entropy of the gas increase, decrease, or remain the same?

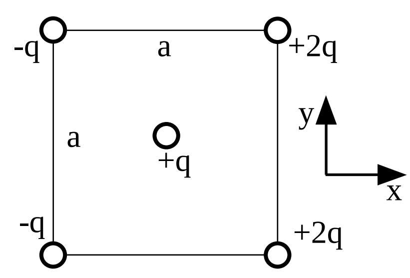
A) Increases. B) Decreases. C) Remains the same.

iv) Two charged particles are fixed in place on a line as shown at right. In which region on the line can a proton be placed such that the proton will remain at rest?



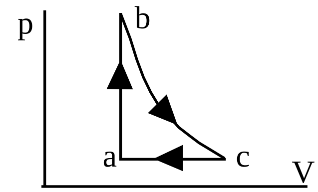
A) Region A B) Region B C) Region C

v) Five charges are arranged at the corners and center of a square as shown at right. What is the direction of the force on the center charge?



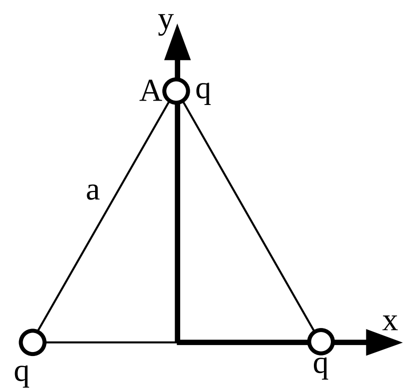
A) Positive *x*-direction. B) Negative *x*-direction. C) Positive *y*-direction. D) Negative *y*-direction

2. (35 points) One mole of an ideal monatomic gas is taken through the cycle *abca* shown at right. Process *bc* is adiabatic. Assume $T_a = 300\ K$, $T_b = 500\ K$, $T_c = 408\ K$, and $R = 8.3\ J/molK$.



a) How much work is done by the gas during the complete cycle?
b) How much heat is transferred to the gas during the stroke *ab* (only part of the cycle).
c) What is the efficiency of this heat engine? (Note that heat is exhausted during the stroke *ca*.)
d) Is the efficiency found in part (c) larger, smaller, or the same as the efficiency of an ideal engine operating between the highest and lowest temperatures that occur in the cycle?
e) Find the entropy change of the gas during the process *ca*.

3. (20 points) Three identical point charges, each with charge $q = 7.5\ \mu C$, are located at the corners of an equilateral triangle of side length $a = 1.5\ m$, as shown at right. Assume that $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$.



a) Find the electrostatic force on charge *A* at the top of the triangle (the one on the *y*-axis).
b) Where must a fourth charge $Q = -5.0\ \mu C$ be placed in order to make the electrostatic force on charge *A* be zero? Determine both the *x* and *y* coordinates of this charge.

4. (20 points) You are having a party and have purchased 200 cans of soda to offer your guests. Each can is made from 20 *g* of aluminum and contains 300 cm^3 of soda. All the cans and their contents have an initial temperature of $T_i = 30^\circ C$ and are placed in a perfectly insulating cooler with some ice (which has an initial temperature of 0˚C). Assume that the soda has the same properties as water and that $c_{Al} = 900\ J/kgK$, $c_{water} = 4000\ J/kgK$, $L_{F,water} = 300\ kJ/kg$, and $\rho_{water} = 1\ g/cm^3$.

a) How much ice is required such that the final equilibrium temperature of the soda (and the entire contents of the cooler) is a refreshing $T_f = 5^\circ C$?
b) How much heat is transferred to each can and its soda in this case?
c) Calculate and discuss the final state of the system (ice, cans, and soda) if twice as much as ice as required in part (a) is used instead.

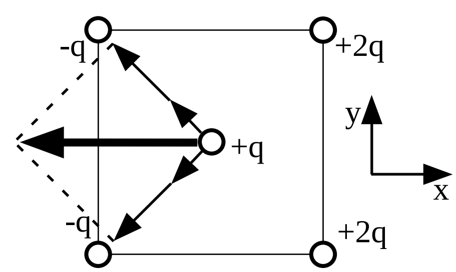
1. i) B For a cyclic process $\Delta E_{\text{int}} = 0$, since the internal energy is a state function (path independent). Thus the first law of thermodynamics implies that $Q = W$ for the cycle. The work done by the gas is just the area of the enclosed path. Both paths are clockwise, so result in positive work done by the gas. The cycle *bcdb* has a larger area, so more work is done, and hence more heat is transferred to the gas.

ii) B During an isobaric process the pressure is constant. The ideal gas law ($pV = nRT$) then implies that T is proportional to V. Thus if the gas is compressed, it must get colder.

iii) A The differential change in entropy is given by $dS = dQ/T$, so the sign of any change in entropy is the same as the sign of the heat transferred. For a constant volume process, the heat is given by $dQ = nC_V dT$ and the ideal gas law ($pV = nRT$) implies that T is proportional to p. Thus an increase in p implies an increase in T and hence a positive heat transfer (*i.e.*, heat goes into the gas). Thus the change in entropy is positive.

iv) B A positively charged proton is repelled by the $+2q$ charge and by the $+3q$ charge. In regions *A* and *C* these repulsions will add to force the proton to infinity. In region *B* the repulsions can cancel at the appropriate place to give no net force.

v) B The diagram at right shows the individual forces and the net force in the negative x-direction.



2. a) The work done in a cycle is the area enclosed by the cycle in the *p-V* diagram, but the adiabat is hard to integrate, so let's use the 1st law of thermodynamics. Around the full cycle, the heat must equal the work, since there is no change in the internal energy. The adiabat has no heat transfer and the other two steps are simply at constant volume and pressure. Thus we get:

$$W = Q = nC_V \Delta T_{ab} + nC_p \Delta T_{ca}$$
$$W = nC_V(T_b - T_a) + nC_p(T_a - T_c)$$
$$W = \tfrac{3}{2}nR(T_b - T_a) + \tfrac{5}{2}nR(T_a - T_c) = nR\left(T_a + \tfrac{3}{2}T_b - \tfrac{5}{2}T_c\right)$$
$$W = 1mol(8.3\,J/molK)\left(300K + \tfrac{3}{2}500K - \tfrac{5}{2}408K\right)$$
$$\boxed{W = 249J}$$

b) Process *ab* is a constant volume process. The heat transferred is found using the appropriate specific heat.

$$Q_{ab} = nC_v \Delta T_{ab}$$
$$Q_{ab} = n\tfrac{3}{2}R(T_b - T_a) = 1mol\,\tfrac{3}{2}(8.3\,J/molK)(500K - 300K)$$
$$\boxed{Q_{ab} = 2490J}$$

c) The efficiency of the cycle is:

$$\varepsilon = \frac{W}{Q_{in}} = \frac{249J}{2490J} = \frac{1}{10}$$
$$\boxed{\varepsilon = 10\%}$$

d) The highest and lowest temperatures in the cycle occur at *b* and *a*, respectively. Thus the efficiency of an ideal engine would be:

$$\varepsilon_{ideal} = 1 - \frac{T_C}{T_H} = 1 - \frac{T_a}{T_b} = 1 - \frac{300K}{500K} = 1 - \frac{3}{5}$$
$$\boxed{\varepsilon_{ideal} = 40\% \quad \Rightarrow \quad \varepsilon < \varepsilon_{ideal}}$$

e) Process *ca* is a constant pressure process, so the heat transferred is $Q = nC_p\Delta T$. The entropy change is thus:

$$\Delta S = \int_c^a \frac{dQ}{T} = \int_{T_c}^{T_a} \frac{nC_p dT}{T} = nC_p \int_{T_c}^{T_a} \frac{dT}{T} = nC_p \ln\frac{T_a}{T_c}$$
$$\Delta S = n\tfrac{5}{2}R\ln\frac{300K}{408K} = 1mol\,\frac{5}{2}(8.3\,J/molK)(-0.307)$$
$$\boxed{\Delta S = -6.38\,J/K}$$

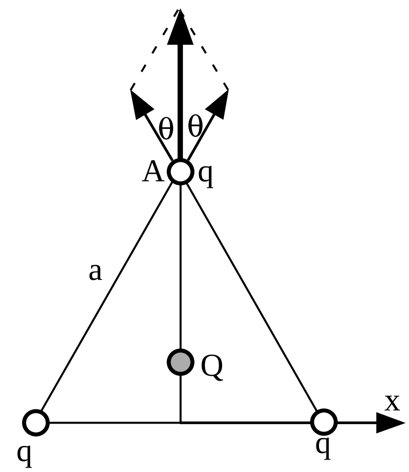
3. a) Since the two charges on the bottom of the triangle are equal and are equidistant from *A*, they produce forces of equal magnitude and the net force is upward along the *y*-axis. The angle θ = 30˚, and the net force is twice the *y*-component of one of the forces.

$$F_y = 2\frac{q^2}{4\pi\varepsilon_0 a^2}\cos\theta = 2\frac{q^2}{4\pi\varepsilon_0 a^2}\cos 30°$$

$$F_y = 2\frac{q^2}{4\pi\varepsilon_0 a^2}\frac{\sqrt{3}}{2} = \frac{q^2\sqrt{3}}{4\pi\varepsilon_0 a^2}$$

$$F_y = \frac{\left(7.5\times10^{-6}C\right)^2\sqrt{3}}{4\pi\left(8.85\times10^{-12}\,C^2/Nm^2\right)(1.5m)^2}$$

$$\boxed{F_y = 0.39N}$$



b) A fourth charge *Q* must produce a force that exactly cancels the force found above. Since *Q* is opposite in sign to *q*, the force will be attractive. Hence *Q* must be placed on the *y*-axis below *q*, as shown above. Let *y* be the vertical position of *Q*. Then the distance to *A* is ($a\cos\theta$-y). Set the new net force equal to zero and solve for *y*:

$$F_y = \frac{q^2\sqrt{3}}{4\pi\varepsilon_0 a^2} - \frac{q|Q|}{4\pi\varepsilon_0 (a\cos\theta - y)^2} = 0$$

$$\frac{q^2\sqrt{3}}{4\pi\varepsilon_0 a^2} = \frac{q|Q|}{4\pi\varepsilon_0 (a\cos\theta - y)^2}$$

$$(a\cos\theta - y)^2 = \frac{q|Q|a^2}{q^2\sqrt{3}}$$

$$a\cos\theta - y = a\sqrt{\frac{|Q|}{q\sqrt{3}}}$$

$$y = a\left(\cos\theta - \sqrt{\frac{|Q|}{q\sqrt{3}}}\right) = 1.5m\left(\frac{\sqrt{3}}{2} - \sqrt{\frac{5\mu C}{7.5\mu C\sqrt{3}}}\right)$$

$$\boxed{\begin{array}{l} y = 0.37m \\ x = 0m \end{array}}$$

4. a) Label the heat transferred to the ice and the resultant melted water that must be heated up Q_{ice} and the heat transferred from the cans and soda Q_{soda}. Since the cooler is insulated, these are the only two heat transfers and they must sum to zero, which means $Q_{\text{ice}} = -Q_{\text{soda}}$. If the final equilibrium temperature of everything is 5˚*C*, then all the ice must melt. Label the mass of the ice m_{ice}.

$$Q_{ice} = m_{ice}L_{F,water} + m_{ice}c_{water}T_f$$

$$Q_{soda} = N\left(m_{can}c_{Al}\Delta T + m_{soda}c_{water}\Delta T\right)$$

$$Q_{ice} = -Q_{soda} \qquad ; \qquad m_{soda} = V_{soda}\rho_{water} = 300cm^3\left(1g/cm^3\right) = 300g$$

$$m_{ice}\left(L_{F,water} + c_{water}T_f\right) = -N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(T_f - T_i\right)$$

$$m_{ice} = \frac{N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(T_i - T_f\right)}{L_{F,water} + c_{water}T_f}$$

$$m_{ice} = \frac{200\left(0.02kg(0.9\,kJ/kgK) + 0.3kg(4\,kJ/kgK)\right)(30°C - 5°C)}{(300\,kJ/kg) + (4\,kJ/kgK)5°C}$$

$$\boxed{m_{ice} = 19kg}$$

b) The heat transferred to each can Q_{each} is given in the above equation for Q_{soda}, since we merely multiplied by *N*, the number of soda cans.

$$Q_{each} = \frac{Q_{soda}}{N} = \frac{N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(T_f - T_i\right)}{N}$$

$$Q_{each} = \left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(T_f - T_i\right)$$

$$Q_{each} = \left(0.02kg(900\,J/kg) + 0.3kg(4000\,J/kgK)\right)(5°C - 30°C)$$

$$\boxed{Q_{each} = -30kJ} \quad < 0 \text{ since heat leaves soda}$$

c) If we use twice as much ice ($2m_{\text{ice}}$), we expect the soda to get roughly twice as cold, but that would be below zero. Thus we expect that not all the ice will melt in this case. Let *x* be the fraction of ice that melts. The final state of the system in such a case must be 0˚*C* for the system to be in equilibrium with unmelted ice.

$$Q_{ice} = x2m_{ice}L_{F,water}$$

$$Q_{soda} = N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(0°C - T_i\right)$$

$$x2m_{ice}L_{F,water} = -N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)\left(0°C - T_i\right)$$

$$x = \frac{N\left(m_{can}c_{Al} + m_{soda}c_{water}\right)T_i}{2m_{ice}L_{F,water}} = \frac{200\left(0.02kg(0.9\,kJ/kgK) + 0.3kg(4\,kJ/kgK)\right)30°C}{2(19kg)(300\,kJ/kg)}$$

$$\boxed{x = 0.64} \quad \boxed{\text{thus 24 } kg \text{ of ice melts and 14 } kg \text{ remains, everything is at } 0˚C}$$