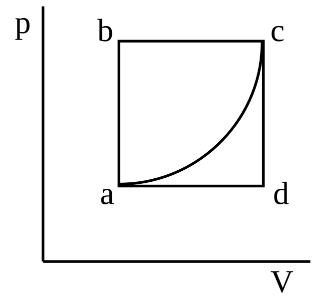
PHYSICS 212 **MIDTERM II** **19 February 2003**

- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (25 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

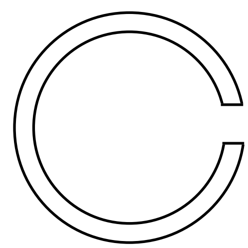
i) An ideal gas is taken through two closed cycle processes. Process A is the cycle *abca*, and process B is the cycle *acda* in the figure at right. For which process is the amount of heat transferred to the gas the largest?

A) Process A B) Process B C) Both the same.



ii) A metallic ring with a small gap is shown at right. The metal has a positive linear expansion coefficient. As the temperature of the ring increases, does the size of the gap increase, decrease, or remain the same?

A) Increases. B) Decreases. C) Remains the same.



iii) An ideal gas undergoes an adiabatic compression. Does the temperature of the gas increase, decrease, or remain the same?

A) Increases. B) Decreases. C) Remains the same.

iv) The volume of an ideal gas is decreased at constant pressure. Does the entropy of the gas increase, decrease, or remain the same?

A) Increases. B) Decreases. C) Remains the same.

v) In an ideal refrigerator, is the heat Q_h transferred to the hot reservoir per cycle less than, greater than, or equal to the work W done per cycle?

A) Less than. B) Greater than. C) The same as.

2. (20 points) A solid steel sphere has a diameter of 8.02 cm at 25˚C. A thin aluminum plate has a circular hole in it with a diameter of 8.00 *cm* at 25˚C. Assume that steel has a linear expansion coefficient $\alpha_S = 10 \times 10^{-6}$/°C and that aluminum has a linear expansion coefficient $\alpha_{Al} = 25 \times 10^{-6}$/°C.

a) To what temperature must the aluminum plate be heated in order for the steel sphere (not heated) to just fit through the plate's hole?

b) If plate and sphere are both heated, what must the common temperature be for the sphere to fit through the hole?

3. (20 points) A 3.0 *kg* aluminum bucket contains 5.0 *kg* of ice, and both have an initial temperature of 0˚C. A hot 6.0 *kg* block of aluminum is added to the bucket and the whole system comes to thermal equilibrium at a final temperature of T_f = 30˚C. Assume c_{Al} = 900 *J/kgK*, c_{water} = 4000 *J/kgK*, $L_{F,water}$ = 300 *kJ/kg*.

a) What was the initial temperature of the aluminum block?

b) After the equilibrium above is reached, more ice is added to the system and a new equilibrium temperature of 0˚C is reached with no ice remaining. How much extra ice was added?

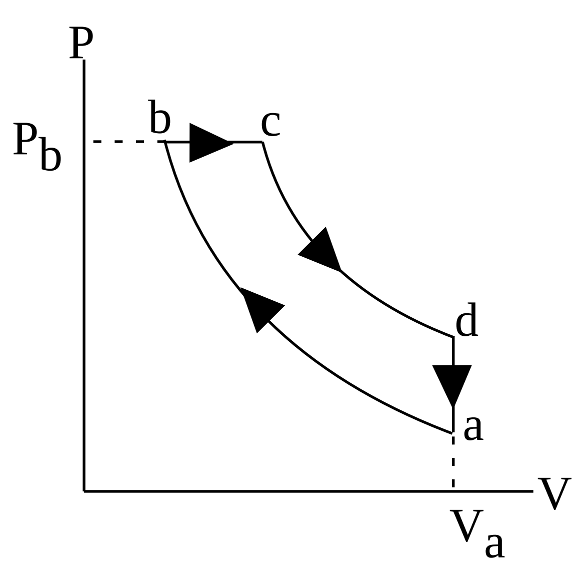
4. (35 points) Five moles of an ideal monatomic gas are taken through the cycle *abcda* shown below. Processes *ab* and *cd* are adiabatic. Assume $P_b = 8.0 \times 10^5$ Pa, $V_b = 0.10\ m^3$, $V_c = 2\ V_b$, $V_a = 10\ V_b$ and $R = 8.3\ J/molK$.

a) How much work is done by the gas during the complete cycle?

b) What is the efficiency of this heat engine?

c) Is the efficiency found in part (b) larger, smaller, or the same as the efficiency of an ideal engine operating between the highest and lowest temperatures that occur in the cycle?

d) Find the entropy change of the gas during the process *bc*.



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1. i) A For a cyclic process $\Delta E_{\text{int}} = 0$, since the internal energy is a state function (path independent). Thus the first law of thermodynamics implies that $Q = W$ for the cycle. The work done by the gas is just the area of the enclosed path. Both paths are clockwise, so result in positive work done by the gas. The cycle *abca* has a larger area, so more work is done, and hence more heat is transferred to the gas.

   ii) A For a material with a positive linear expansion coefficient, each linear dimension increases as the temperature increases. Thus the circumference of the ring increases, as do all parts of the circumference including the part that is there and the part that is not. The gap increases in the same manner that the missing piece itself would expand.

   iii) A During an adiabatic process $Q = 0$. This implies that $\Delta E_{\text{int}} = -W$. In any compression, the gas does negative work, so in this case the internal energy must increase. Since $E_{\text{int}} = nC_{\text{V}}T$, the temperature must increase. That is, the work done on the gas increases the internal energy of the gas.

   iv) B The differential change in entropy is given by $dS = dQ/T$, so the sign of any change in entropy is the same as the sign of the heat transferred. For a constant pressure process, the heat is given by $dQ = nC_{\text{p}}dT$ and the ideal gas law ($pV = nRT$) implies that T is proportional to V. Thus a decrease in V implies a decrease in T and hence a negative heat transfer (*i.e.*, heat leaves the gas). Thus the change in entropy is negative.

   v) B In a refrigerator, the heat in plus the work done on the system equals the heat out: $Q_{in} + W = Q_{out}$, or $Q_c + W = Q_h$. Thus the heat transferred to the hot reservoir is larger than the work done.

2. a) In the first case only the plate expands. The hole also expands in the same proportions. Equate the expanded hole to the sphere to see when they will just fit:

$$d_{Al} = d_{Al,i} + \alpha_{Al} d_{Al,i} \Delta T = d_{S,i}$$

$$\Delta T = \frac{d_{S,i} - d_{Al,i}}{\alpha_{Al} d_{Al,i}}$$

$$T_f - T_i = \frac{d_{S,i} - d_{Al,i}}{\alpha_{Al} d_{Al,i}}$$

$$T_f = T_i + \frac{d_{S,i} - d_{Al,i}}{\alpha_{Al} d_{Al,i}}$$

$$T_f = 25°C + \frac{8.02cm - 8.00cm}{25 \times 10^{-6}(8.00cm)}$$

$$\boxed{T_f = 125°C}$$

b) In the second case both sphere and hole expand, but the hole expands more since it has a larger expansion coefficient.

$$d_{Al} = d_{Al,i} + \alpha_{Al} d_{Al,i} \Delta T = d_{S,i} + \alpha_S d_{S,i} \Delta T$$

$$\left(\alpha_{Al} d_{Al,i} - \alpha_S d_{S,i}\right) \Delta T = d_{S,i} - d_{Al,i}$$

$$T_f - T_i = \frac{d_{S,i} - d_{Al,i}}{\alpha_{Al} d_{Al,i} - \alpha_S d_{S,i}}$$

$$T_f = T_i + \frac{d_{S,i} - d_{Al,i}}{\alpha_{Al} d_{Al,i} - \alpha_S d_{S,i}}$$

$$T_f = 25°C + \frac{8.02cm - 8.00cm}{25 \times 10^{-6}(8.00cm) - 10 \times 10^{-6}(8.02cm)}$$

$$\boxed{T_f = 192°C}$$

3. a) To find the initial temperature of the block, equate the heat transferred to the bucket and ice & water to the heat transferred from the block, taking account of the minus sign.

$$Q_{bucket} + Q_{ice+water} = -Q_{block}$$
$$Q_{bucket} = m_{bucket}c_{Al}(T_f - T_i)$$
$$Q_{ice+water} = m_{ice}L_{F,water} + m_{water}c_{water}(T_f - T_i)$$
$$Q_{block} = m_{block}c_{Al}(T_f - T_{i,block})$$
$$m_{bucket}c_{Al}(T_f - T_i) + m_{ice}L_{F,water} + m_{water}c_{water}(T_f - T_i) = -m_{block}c_{Al}(T_f - T_{i,block})$$
$$\frac{(m_{bucket}c_{Al} + m_{water}c_{water})(T_f - T_i) + m_{ice}L_{F,water}}{m_{block}c_{Al}} = -T_f + T_{i,block}$$
$$T_{i,block} = T_f + \frac{(m_{bucket}c_{Al} + m_{water}c_{water})(T_f - T_i) + m_{ice}L_{F,water}}{m_{block}c_{Al}}$$
$$T_{i,block} = 30°C + \frac{[3kg(900\,J/kgK) + 5kg(4000\,J/kgK)]30°C + 5kg(300\,kJ/kg)}{6kg(900\,J/kgK)}$$
$$\boxed{T_{i,block} = 434°C}$$

b) Now equate the heat that is required to melt the new ice with the amount of heat that is transferred from the bucket, block, and water as they are cooled from T_{f1} = 30˚C to T_{f2} = 0˚*C*.

$$Q_{ice,melt} = m_{ice2}L_{F,water}$$
$$Q_{Al,water} = (m_{bucket+block}c_{Al} + m_{water}c_{water})(T_{f2} - T_{f1})$$
$$Q_{ice,melt} = -Q_{Al,water}$$
$$m_{ice2}L_{F,water} = -(m_{bucket+block}c_{Al} + m_{water}c_{water})(T_{f2} - T_{f1})$$
$$m_{ice2} = \frac{(m_{bucket+block}c_{Al} + m_{water}c_{water})(T_{f1} - T_{f2})}{L_{F,water}}$$
$$m_{ice2} = \frac{(9kg(0.9\,kJ/kgK) + 5kg(4\,kJ/kgK))30°C}{300\,kJ/kg}$$
$$\boxed{m_{ice2} = 2.81\ kg}$$

4. a) The work done in a cycle is the area enclosed by the cycle in the *p-V* diagram, but the adiabat is hard to integrate, so we use the 1st law of thermodynamics. Around the full cycle, the heat must equal the work, since there is no change in the internal energy. The adiabats have no heat transfer and the other two steps are simply at constant volume and constant pressure. To find the temperatures, relate the points using the ideal gas law and the adiabat definition.

$$W = Q_{net} = Q_{bc} + Q_{da} = nC_P\Delta T_{bc} + nC_V\Delta T_{da}$$

$$W = \tfrac{5}{2}nR(T_c - T_b) + \tfrac{3}{2}nR(T_a - T_d) = \frac{nR}{2}\left(5(T_c - T_b) + 3(T_a - T_d)\right)$$

$$T_b:\quad P_bV_b = nRT_b \quad\Rightarrow\quad T_b = \frac{P_bV_b}{nR} = \frac{8\times10^5\,Pa\left(0.1m^3\right)}{5mol(8.3\,J/molK)} = 1928K$$

$$bc:\quad P_c = P_b\ ,\ V_c = 2V_b \quad\Rightarrow\quad T_c = \frac{P_cV_c}{nR} = \frac{P_b2V_b}{nR} = 2T_b = 3855K$$

$$cd:\quad P_cV_c^\gamma = P_dV_d^\gamma \quad\Rightarrow\quad P_d = P_c\left(\frac{V_c}{V_d}\right)^\gamma = P_b\left(\frac{2V_b}{10V_b}\right)^\gamma \quad\Rightarrow\quad T_d = \frac{P_dV_d}{nR} = \frac{P_b10V_b}{nR}\left(\tfrac{1}{5}\right)^{\frac{5}{3}} = 10\left(\tfrac{1}{5}\right)^{\frac{5}{3}}T_b = 1319K$$

$$ab:\quad P_aV_a^\gamma = P_bV_b^\gamma \quad\Rightarrow\quad P_a = P_b\left(\frac{V_b}{V_a}\right)^\gamma = P_b\left(\frac{V_b}{10V_b}\right)^\gamma \quad\Rightarrow\quad T_a = \frac{P_aV_a}{nR} = \frac{P_b10V_b}{nR}\left(\tfrac{1}{10}\right)^{\frac{5}{3}} = 10\left(\tfrac{1}{10}\right)^{\frac{5}{3}}T_b = 415K$$

$$W = \frac{5mol(8.3\,J/molK)}{2}\left(5(3855K - 1928K) + 3(415K - 1319K)\right)$$

$$\boxed{W = 144kJ}$$

b) Heat is transferred into the gas during the process *bc*, which is a constant pressure process.

$$Q_{bc} = nC_P\Delta T_{bc}$$

$$Q_{bc} = n\tfrac{5}{2}R(T_c - T_b) = 5mol\,\tfrac{5}{2}(8.3\,J/molK)(3855K - 1928K) = 200kJ$$

$$\varepsilon = \frac{W}{Q_{in}} = \frac{W}{Q_{bc}} = \frac{144J}{200J}$$

$$\boxed{\varepsilon = 72\%}$$

c) The highest and lowest temperatures in the cycle occur at *c* and *a*, respectively. Thus the efficiency of an ideal engine would be:

$$\varepsilon_{ideal} = 1 - \frac{T_C}{T_H} = 1 - \frac{T_a}{T_c} = 1 - \frac{415K}{3855K}$$

$$\boxed{\varepsilon_{ideal} = 89\% \qquad\Rightarrow\qquad \varepsilon < \varepsilon_{ideal}}$$

d) Process *bc* is a constant pressure process, so the heat transferred is $Q = nC_P\Delta T$. The entropy change is thus:

$$\Delta S = \int_b^c \frac{dQ}{T} = \int_{T_b}^{T_c} \frac{nC_PdT}{T} = nC_p\int_{T_b}^{T_c}\frac{dT}{T} = nC_p\ln\frac{T_c}{T_b} = nC_p\ln\frac{2T_b}{T_b}$$

$$\Delta S = n\tfrac{5}{2}R\ln 2 = 5mol\frac{5}{2}(8.3\,J/molK)(0.693)$$

$$\boxed{\Delta S = 71.9\,J/K}$$