PHYSICS 212 **MIDTERM I** **26 January 2000**

- Exam is closed book, closed notes. Use only the provided formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (20 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

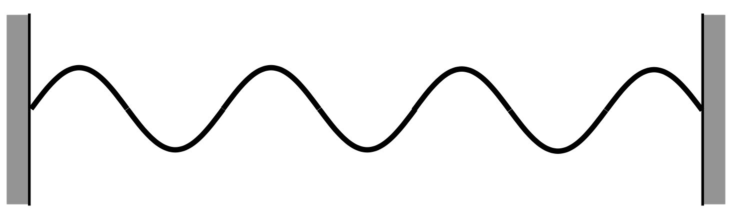
i) A boat with an anchor on board floats in the ocean. The anchor is tied to a rope and slowly lowered over the side of the boat into the water. As the anchor goes into the water with the rope still taut, does the boat move upward, move downward, or remain the same with respect to the water level in the ocean?

A) Upward. B) Downward. C) Remains the same.

ii) When mass m_1 is hung from spring 1 and a smaller mass m_2 is hung from spring 2, the two masses (both at rest) cause the two springs to be stretched by the same distance. Which system will have the larger frequency of oscillation?

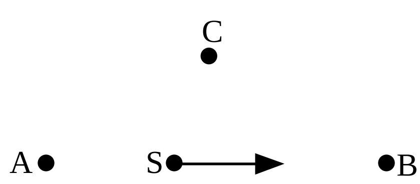
A) System 1. B) System 2. C) They will have the same frequency.

iii) A standing wave of frequency f is produced on a string that is stretched between two fixed supports, as shown below. The tension in the string is then increased and a new standing wave is produced in the string with the same frequency f. Does the new standing wave have more nodes, fewer nodes, or the same number of nodes as the original standing wave?



A) More nodes. B) Fewer nodes. C) The same number of nodes.

iv) Three stationary observers *A*, *B*, and *C* are listening to a moving source of sound (S), as shown in the diagram. Which observer detects the largest sound frequency?



v) Two organ pipes, each with two open ends, are excited in their respective first harmonic modes. They are slightly different in length and so produce a beat note. The beat frequency becomes smaller when organ pipe A is made longer while organ pipe B is kept constant in length. Which organ pipe is longer than the other?

A) Organ pipe A B) Organ pipe B

2. (20 points) A solid cube of side length a = 5.0 *cm* and density ρ_{cube} = 0.8 g/cm^3 is floating in water (density ρ_W = 1.0 g/cm^3). Assume g = 10 m/s^2.

a) How far below the water level is the bottom surface of the cube?
b) A layer of oil (density ρ_{oil} = 0.5 g/cm^3) is added to the water such that the oil completely covers the cube. Now how far below the water level is the bottom surface of the cube?

3. (20 points) A cylindrical water storage tank of height h = 5.0 *m* is completely filled with water and rests on the floor. A small hole is punched in the side of the tank a distance d = 1.0 *m* above the floor. Assume g = 10 m/s^2, and the tank's surface area is much larger than the hole.

a) How far horizontally from the edge of the water tank does the stream of water from the hole hit the floor?
b) How far from the floor should the hole be if the stream of water is to land as far as possible horizontally from the tank?

4. (20 points) Organ pipe A, with both ends open, has a fundamental frequency of 250 *Hz*. The fifth harmonic of organ pipe B, with one end closed and one end open, has the same frequency as the second harmonic of pipe A. Assume the speed of sound is v_s = 340 m/s.

a) How long is pipe A?
b) How long is pipe B?

5. These two problems are not related, but please work both of them on the same page.

a) (10 points) A mass of 2 *kg* connected to a massless spring of spring constant 20 N/m oscillates on a horizontal, frictionless surface. The amplitude of the motion is 4 *m*. Find the period of oscillation of the mass and the speed of the mass when the displacement from the relaxed position of the spring is 2 *m*.

b) (10 points) The equation of a transverse wave traveling along a string is $y = (3.0\ mm)\sin[(25\ m^{-1})\,x + (500\ s^{-1})\,t]$. Find the amplitude, frequency, velocity, wavelength, and direction of travel (positive or negative x-direction) of the wave.

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1. i) A Before the anchor enters the water, the boat must displace enough water to support the weight of the boat and of the anchor. When the anchor enters the water, there is a new buoyant force acting directly on the anchor, so less buoyant force is needed on the boat. Hence the boat displaces less water and so moves upward.

   ii) C If we call the distance each spring stretches x, then we must have: $k_1 x = m_1 g$ and $k_2 x = m_2 g$. The resonant frequency is proportional to $\sqrt{k_i / m_i}$ in each case. For both cases the ratio $k_i/m_i = g/x$, so the two frequencies must be the same.

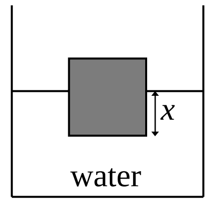
   iii) B The speed of waves in a string is $v = \sqrt{\tau/\mu}$, so when the tension increases, the wave speed will increase. A standing wave will occur when $L = n\lambda/2$. Since the wavelength is given by $\lambda = v/f$ and the frequency remains the same, an increased wave speed implies a longer wavelength. Thus there must be fewer nodes in the standing wave.

   iv) B The Doppler shift causes the detected frequency to increase when the source approaches the receiver.

   v) B The lowest harmonic of an organ pipe with two open ends corresponds to $L = \lambda/2$, giving a frequency of $f = v/\lambda = v/2L$. When organ pipe A is made longer, its frequency will decrease. Since this decreases the beat note, the frequency of organ pipe B must be less than the frequency of organ pipe A. A smaller frequency (for B) implies a longer tube.

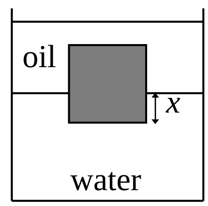
2. a) Let the depth of the cube in the water be *x*. Archimedes' principle states that the buoyant force is equal to the weight of the displaced fluid. Since the cube is floating, the buoyant force must equal the weight of the cube.

$$F_{buoyant} = m_{cube} g$$
$$\rho_w g a^2 x = \rho_{cube} g a^3$$
$$\rho_w x = \rho_{cube} a$$
$$x = \frac{\rho_{cube}}{\rho_w} a = \frac{0.8\, g/cm^3}{1.0\, g/cm^3} 5.0 cm$$
$$\boxed{x = 4.0 cm}$$



b) In this case, there are two types of fluid, and we must include the total weight of displaced fluids.

$$F_{buoyant} = m_{cube} g$$
$$\rho_w g a^2 x + \rho_{oil} g a^2 (a - x) = \rho_{cube} g a^3$$
$$\rho_w x + \rho_{oil}(a - x) = \rho_{cube} a$$
$$(\rho_w - \rho_{oil}) x = (\rho_{cube} - \rho_{oil}) a$$
$$x = \frac{\rho_{cube} - \rho_{oil}}{\rho_w - \rho_{oil}} a$$
$$x = \frac{0.8\, g/cm^3 - 0.5\, g/cm^3}{1.0\, g/cm^3 - 0.5\, g/cm^3} 5.0 cm$$
$$\boxed{x = 3.0 cm}$$



3. a) Apply Bernoulli's equation at the top of the water storage tank and at the leaky hole. Pressure is atmospheric at both points, and since the tank is so large, the velocity of the top surface is zero. Define the floor to be at $y = 0$.

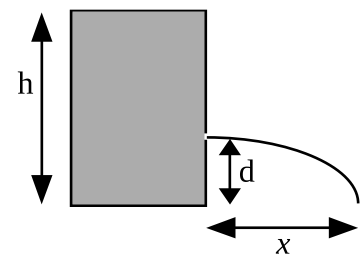
$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_0 + 0 + \rho g h = p_0 + \tfrac{1}{2}\rho v^2 + \rho g d$$

$$\tfrac{1}{2}\rho v^2 = \rho g(h - d)$$

$$v = \sqrt{2g(h-d)} = \sqrt{2(10\,m/s^2)(4.0m)} = 8.95\,m/s$$

The horizontal distance traveled is determined by kinematics



$$y = y_0 + v_{oy}t + \tfrac{1}{2}at^2$$

$$0 = d - \tfrac{1}{2}gt^2$$

$$t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(1.0m)}{10.0\,m/s^2}} = 0.447s$$

$$x = vt = \sqrt{2g(h-d)}\sqrt{\frac{2d}{g}}$$

$$x = 2\sqrt{d(h-d)} = 2\sqrt{1.0m(4.0m)}$$

$$\boxed{x = 4.0m}$$

b) This question asks us to maximize the horizontal distance x with respect to the variable d. To make the notation clearer, change d to the variable y. Thus we must find the maximum of the function $x(y)$. We do this by differentiating and setting the derivative to zero.

$$x(y) = 2\sqrt{y(h-y)} = 2[y(h-y)]^{\frac{1}{2}}$$

$$\frac{dx}{dy} = [y(h-y)]^{-\frac{1}{2}}[y(-1) + (h-y)]$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{y(h-y)}}[h - 2y] = 0$$

$$h - 2y = 0$$

$$y = \frac{h}{2} = \frac{5.0m}{2}$$

$$\boxed{y = 2.5m}$$

For this value of the height of the hole, the stream lands

$$x = 2\sqrt{y(h-y)} = 2\sqrt{2.5m(2.5m)}$$

$\boxed{x = 5.0m}$ away from the tank.

4. a) An organ pipe with two open ends (A) has a half-wave fundamental and all integer harmonics, $L_A = n_A\lambda/2$. Thus for pipe A we get:

$$f_A = \frac{v}{\lambda} = \frac{v}{\frac{2L_A}{n_A}} = n_A \frac{v}{2L_A}, \quad n_A = 1, 2, 3, \ldots$$

$$f_{A,1} = \frac{v}{2L_A}$$

$$L_A = \frac{v}{2f_{A,1}} = \frac{340\,m/s}{2(250\,Hz)}$$

$$\boxed{L_A = 0.68m}$$

b) An organ pipe with one end open and one end closed (B) has a quarter-wave fundamental and odd integer harmonics, $L_B = n_B\lambda/4$. Thus for pipe B we get:

$$f_B = \frac{v}{\lambda} = \frac{v}{\frac{4L_B}{n_B}} = n_B \frac{v}{4L_B}, \quad n_B = 1, 3, 5, \ldots$$

$$f_{B,5} = f_{A,2}$$

$$5\frac{v}{4L_B} = 2\frac{v}{2L_A}$$

$$L_B = \tfrac{5}{4}L_A = \tfrac{5}{4}0.68m$$

$$\boxed{L_B = 0.85m}$$

5. a) The period is found by first finding the angular frequency of oscillation

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20\,N/m}{2kg}} = 3.16\,rad/s$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2kg}{20\,N/m}}$$

$$\boxed{T = 2s}$$

Assume the standard form for the oscillation displacement and then differentiate to find the velocity.

$$x = x_m \cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$x = 2m \quad \Rightarrow \quad 2m = 4m\cos(\omega t + \phi) \quad \Rightarrow \quad \cos(\omega t + \phi) = \frac{1}{2}$$

$$\omega t + \phi = \arccos\left(\tfrac{1}{2}\right) = 60^\circ \text{ or } \tfrac{\pi}{3} \text{ radians}$$

$$v = -\omega x_m \sin(\omega t + \phi) = -(3.16\,rad/s)(4m)\sin\left(\tfrac{\pi}{3}\right)$$

$$\boxed{v = 11\,m/s}$$

The sign is neglected since we could be moving either direction, that's why only the speed was asked for.

b) The general equation of a wave is

$$y = y_m \sin(kx \mp \omega t)$$

Thus we can determine the following:

$$\boxed{y_m = 3.0mm}$$

$$f = \frac{\omega}{2\pi} = \frac{500s^{-1}}{2\pi}$$

$$\boxed{f = 80Hz}$$

$$v = \frac{\omega}{k} = \frac{500s^{-1}}{25m^{-1}}$$

$$\boxed{v = 20\,m/s}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{25m^{-1}}$$

$$\boxed{\lambda = 0.25m}$$

The direction of travel of the wave is determined by examining the phase ($kx+\omega t$). What must x do as t increases to keep the phase constant? In this case, x must decrease as t increases, so wave travels in the $\boxed{\text{negative}}$ x-direction.