- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (20 points)   For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

i) A boat made entirely of wood floats in a large swimming pool. Does the water level in the pool move upward, move downward, or remain the same when the boat is disassembled and all the wooden pieces are floating individually in the pool?

A) Upward.   B) Downward.   C) Remains the same.

ii) Two strings of unequal linear mass densities are tied together and stretched between two supports. A standing wave is produced on the string, with a node at the knot, as shown below. Which string has the
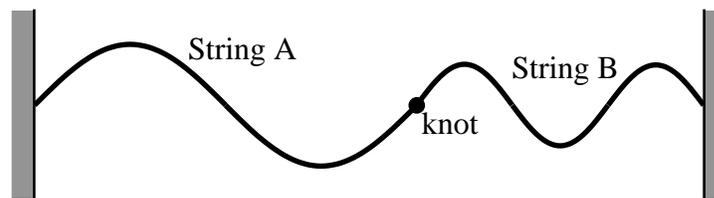
greater linear mass density?

iii) A skydiver jumps from an airplane while the pilot is playing an F note on his harmonica. As the skydiver falls, she continues to hear the harmonica. Is the frequency she hears greater than, less than, or equal to the frequency of an F note?

A) Greater than.   B) Less than.   C) Equal to.

iv) The equations below describe three waves. Which wave has the largest speed?

A) $y_A = 2 \cos(3x - 4t)$ B) $y_B = 3 \cos(4x + 3t)$ C) $y_C = 4 \cos(2x - 3t)$

v) A stretched string of length L is made to oscillate at its lowest resonant frequency. The lowest harmonic of an organ pipe with one end open and one end closed has the same resonant wavelength. How long is the organ pipe?

A) $L/4$ B) $L/2$ C) L D) $2L$

2. (20 points) A tall, thin cylindrical jar has length $L = 8.0$ cm, cross-sectional area $A = 4.0$ cm^2, and mass $m_{jar} = 6.0$ g. The jar floats in water of density $\rho_W = 1.0$ g/cm^3, but can be made to sink by adding a number of small nuts, each of which has a mass $m_{nut} = 5.0$ g.

a) What is the minimum number of nuts that must be added to make the jar sink in the water?

b) With this minimum number of nuts, the jar floats in a fluid of density $\rho_{fluid} = 1.5$ g/cm^3. How far above the fluid level is the top surface of the floating jar? Assume the jar floats with the length L vertical.

3. (20 points) A water storage tank of height $h = 6.2$ m is completely filled with water and rests on the floor. A small hole of cross-sectional area $A = 2.0$ cm^2 is punched in the side of the tank a distance $d = 3.0$ m above the floor. A bucket of volume $V = 4$ L is placed on the floor and collects all the water leaving the tank. Assume the tank's surface area is much larger than the hole.

a) Find the speed of the water as it leaves the hole.

b) How long does it take to fill the initially empty bucket?

4. (20 points) Organ pipe A, with both ends open, has a fundamental frequency of 100 Hz. The third harmonic of organ pipe B, with one end closed and one end open, has the same frequency as the second harmonic of pipe A. Assume the speed of sound is $v_s = 340$ m/s.

a) How long is pipe A?

b) How long is pipe B?

5. (20 points) The equation of a transverse wave traveling along a string is

$$y(x,t) = (4.00mm)\sin\left[\left(33m^{-1}\right)x + \left(660s^{-1}\right)t\right]$$

a) Find the amplitude, frequency, velocity, wavelength, and direction of travel (positive or negative x–direction) of the wave.

b) Find the transverse displacement, transverse velocity, and transverse acceleration of the string at a position $x = 2.0$ m at a time $t = 1.3$ s.

1. i)   C   When the boat is floating, the displaced water has the same weight as the total weight of the boat. When the individual pieces float, the same weight must be supported, so the same amount of water must be displaced.

   ii)   B   The speed of waves in a string is $v = \sqrt{T/\mu}$. The tension in each string must be the same since they are tied together. The frequency of the wave must also be the same or the strings would not form a continuous wave at the knot. The wavelengths differ as shown. The wavelength is given by $\lambda = v/f = \sqrt{T/\mu}/f$, such that longer wavelength corresponds to smaller density μ. String B has a shorter wavelength, so must have a greater mass density.

   iii)   B   The Doppler shift causes the detected frequency to decrease when the receiver recedes from the source.

   iv)   C   The waves are written in the form $y = y_{max} \cos(kx \pm \omega t)$. The speed of a wave is given by $v = \omega/k$. This ratio is largest for wave C. The sign tells us which way the wave travels.

   v)   B   The lowest harmonic of a stretched string corresponds to $L = \lambda/2$. The lowest harmonic of a closed organ pipe corresponds to $L_{organ} = \lambda/4$. If the wavelengths are equal, then $L_{organ} = \lambda/4 = 2L/4 = L/2$.

2. a) In order for the jar to sink, the weight of the jar and the nuts must be greater than the buoyant force of the fluid. Archimedes' principle states that the buoyant force is equal to the weight of the displaced fluid. In this case, the complete volume of the jar will be displaced when it sinks. Let N = the number of nuts in the jar. We must solve for the smallest integer that satisfies the sinking condition.

$$m_{total}g > B$$

$$\left(m_{jar} + Nm_{nut}\right)g > \rho_w Vg$$

$$m_{jar} + Nm_{nut} > \rho_w AL$$

$$Nm_{nut} > \rho_w AL - m_{jar}$$

$$N > \frac{\rho_w AL - m_{jar}}{m_{nut}}$$

$$N > \frac{1.0\,g/cm^3\left(4.0cm^2\right)(8cm) - 6.0g}{5.0g}$$

$$N > 5.2$$

$$\boxed{N_{min} = 6}$$

b) The jar floats in the new fluid. Let x be the distance from the top of the fluid to the top of the jar. Since the jar floats, the weight of the jar equals the buoyant force.

$$B = m_{total}g$$

$$\rho_{fluid}gA(L - x) = \left(m_{jar} + N_{min}m_{nut}\right)g$$

$$L - x = \frac{m_{jar} + N_{min}m_{nut}}{\rho_{fluid}A}$$

$$x = L - \frac{m_{jar} + N_{min}m_{nut}}{\rho_{fluid}A}$$

$$x = 8.0cm - \frac{6.0g + 6(5.0g)}{1.5\,g/cm^3\left(4.0cm^2\right)}$$

$$\boxed{x = 2.0cm}$$
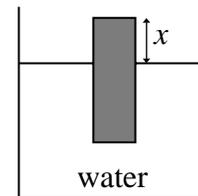
3. a) Apply Bernoulli's equation at the top of the water storage tank and at the leaky hole. Pressure is atmospheric at both points, and since the tank is so large, the velocity of the top surface is zero. Define the floor to be at $y = 0$.

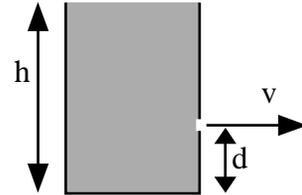$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_0 + 0 + \rho g h = P_0 + \tfrac{1}{2}\rho v^2 + \rho g d$$

$$\tfrac{1}{2}\rho v^2 = \rho g(h - d)$$

$$v = \sqrt{2g(h - d)}$$

$$v = \sqrt{2(10\,m/s^2)(6.2m - 3.0m)}$$

$$\boxed{v = 8\,m/s}$$

b) The volume flow rate R times the time t to fill the bucket must equal the volume V of the bucket.

$$R = Av$$

$$V = Rt = Avt$$

$$t = \frac{V}{Av} = \frac{4 \times 10^3\,cm^3}{2.0cm^2\left(8 \times 10^2\,cm/s\right)}$$

$$\boxed{t = 2.5s}$$

4. a) An organ pipe with two open ends (A) has a half-wave fundamental and all integer harmonics. Thus for pipe A we get:

$$f_A = n\frac{v}{2L_A}, \quad n = 1, 2, 3, \ldots$$

$$f_{A,1} = \frac{v}{2L_A}$$

$$L_A = \frac{v}{2f_{A,1}} = \frac{340\,m/s}{2(100\,Hz)}$$

$$\boxed{L_A = 1.7m}$$

b) An organ pipe with one end open and one end closed (B) has a quarter-wave fundamental and odd integer harmonics. Thus for pipe B we get:

$$f_B = \frac{v}{\lambda} = n\frac{v}{4L_B}, \quad n = 1, 3, 5, \ldots$$

$$f_{B,3} = f_{A,2}$$

$$3\frac{v}{4L_B} = 2\frac{v}{2L_A}$$

$$L_B = \tfrac{3}{4}L_A = \tfrac{3}{4}1.7m$$

$$\boxed{L_B = 1.3m}$$

5.   a)   The general equation of a wave is
$$y(x,t) = A\sin(kx \mp \omega t)$$
For our wave we have:
$$y(x,t) = (4.00mm)\sin\left[\left(33m^{-1}\right)x + \left(660s^{-1}\right)t\right]$$
Thus we can determine the following:

$$\boxed{A = 4.00mm}$$
$$f = \frac{\omega}{2\pi} = \frac{660s^{-1}}{2\pi}$$
$$\boxed{f = 105Hz}$$
$$v = \frac{\omega}{k} = \frac{660s^{-1}}{33m^{-1}}$$
$$\boxed{v = 20\,m/s}$$
$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{33m^{-1}}$$
$$\boxed{\lambda = 0.19m}$$

The direction of travel of the wave is determined by examining the phase ($kx+\omega t$). What must x do as t increases to keep the phase constant? In this case, x must decrease as t increases, so wave travels in the $\boxed{\text{negative}}$ x-direction.

b)   The displacement is found by evaluating the wave equation at the desired point and time.

$$y(x,t) = A\sin(kx + \omega t)$$
$$y(2.0m,1.3s) = (4.0mm)\sin\left[\left(33m^{-1}\right)2.0m + \left(660s^{-1}\right)1.3s\right]$$
$$y(2.0m,1.3s) = (4.0mm)\sin(924rad)$$
$$\boxed{y(2.0m,1.3s) = 1.45mm}$$

The transverse velocity is found by differentiating the position:

$$v_y(x,t) = \frac{\partial y}{\partial t} = \omega A\cos(kx + \omega t)$$
$$v_y(2.0m,1.3s) = \left(660s^{-1}\right)(4.0mm)\cos\left[\left(33m^{-1}\right)2.0m + \left(660s^{-1}\right)1.3s\right]$$
$$\boxed{v_y = 2.46\,m/s}$$

The transverse acceleration is found by differentiating the velocity:

$$a_y(x,t) = \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx + \omega t)$$
$$a_y(2.0m,1.3s) = -\left(660s^{-1}\right)^2(4.0mm)\sin\left[\left(33m^{-1}\right)2.0m + \left(660s^{-1}\right)1.3s\right]$$
$$\boxed{a_y = -6.33\times10^2\,m/s^2}$$