- Exam is closed book, closed notes. Use only the provided formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- **Show all your work and explain your reasoning** (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.
- Charges labeled +q are positive and those labeled -q are negative.

1. (35 points)   For each of these multiple choice questions, indicate the correct response (A, B, C, or D) on the page for problem 1 in your exam booklet.

i)   A boat with an anchor on board floats in a large swimming pool. Does the water level in the pool move upward, move downward, or remain the same when the anchor is thrown overboard and sinks to and rests on the bottom of the pool.
   A) Upward.   B) Downward.   C) Remains the same.

ii) An ideal gas undergoes an adiabatic expansion. Does the temperature of the gas increase, decrease, or remain the same?
   A) Increases.   B) Decreases.   C) Remains the same.

iii) An infinite parallel plate capacitor is connected to a battery of potential difference V. A proton is released from rest inside the capacitor and accelerates toward one of the plates. After the proton has traveled a distance d, its velocity is proportional to what factor?
   A) d B) d^2 C) $d^{1/2}$ D) d^{-1}

iv) A stretched string of length L is made to oscillate at its lowest resonant frequency. The lowest harmonic of an organ pipe with one end open and one end closed has the same resonant wavelength. How long is the organ pipe?
   A) $L/4$ B) $L/2$ C) L D) $2L$

v)   A plastic rod is bent into a quarter circle and has a charge q distributed uniformly along its length. More identical quarter circle arcs are added until the circle is complete, producing the four possible arrangements shown at right. In all four cases, each quarter circular arc has the same charge q. In which arrangement is the magnitude of the electric field at the origin the largest?
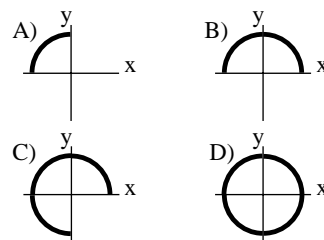

vi) Consider three different arrangements of two charges as shown below. If the electric potential is defined to be zero at infinity in each case, then for which arrangement can one find points other than at infinity at which the electric potential is zero?

A)   -q   +q   B)   -q   -q   C)   +q   +q

vii) Three identical light bulbs are connected in the circuit shown at right. When light bulb 2 is removed from the circuit, does the brightness of bulb 1 increase, decrease, or remain the same?
   A) Increases.   B) Decreases.   C) Remains the same.
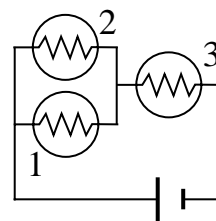
2. (30 points) One mole of an ideal monatomic gas is taken through the cycle *abcda* shown at right. Assume $V_0 = 0.25\ m^3$, $p_0 = 1.00 \times 10^5$ Pa, and $R = 8.3\ J/molK$.

   a) How much work is done by the gas during the complete cycle?
   b) How much heat is transferred to the gas during the stroke *abc* (only part of the cycle).
   c) What is the efficiency of this heat engine? (Note that heat is exhausted during the stroke *cda*.)
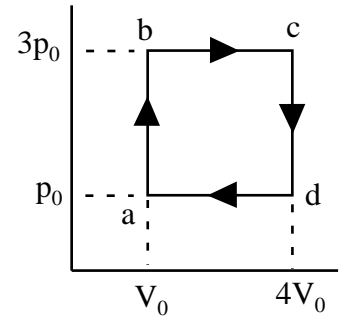
3. (25 points) Five identical light bulbs are connected in a circuit as shown at right. Each light bulb dissipates 20 *W* of electrical power when a 10 *V* electrical potential difference is placed across the bulb.

   a) Find the current through light bulbs 1 and 5 when the switch is open.
   b) After the switch is closed, does bulb 1 get brighter, dimmer, or remain the same, and why?
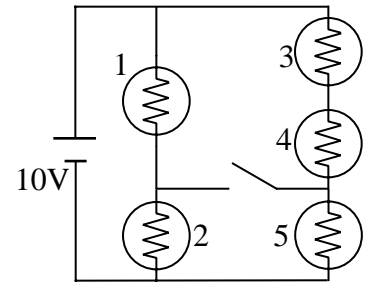
4. (30 points) Two capacitors ($C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$) are connected in a circuit as shown at right. (Switch is open for (a) and (b)).

   a) Find the charge on each capacitor.
   b) Find the charge on each capacitor when a dielectric material of dielectric constant $\kappa = 3$ is inserted in capacitor 2.
   c) With the dielectric in place, close the switch to connect the resistor $R = 5\ \Omega$. Find the current through the resistor immediately after closing the switch and then a long time later.
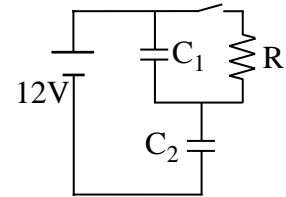
5. (30 points) A thin plastic rod of length *L* lies along the positive *x*-axis, with one end a distance *d* from the origin as shown at right. A charge *q* is uniformly distributed along the length of the rod. Find the electric field at the origin, in terms of the parameters given above and any other physical constants required.
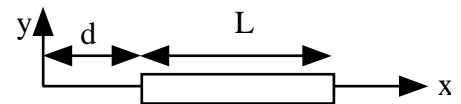
6. (30 points) A wire with uniform charge density $\lambda = 4.0\ \mu C/m$ is bent into the shape shown at right. The radius of the semicircular part is $R = 5.0\ m$ and the lengths of the straight sections are $L = 3.0\ m$. Assuming that the electric potential is zero at infinity, determine the electric potential at the point *O* at the center of the semicircle. Assume that $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$.

7. (20 points) A solid cube (0.300 *m* on a side) of density $\rho_{cube} = 3.00 \times 10^3\ kg/m^3$ is suspended by a string. The cube is lowered into a bucket of water until it is completely below the water level, but does not contact the bucket. The bucket and water together have a mass of 8.00 *kg* and rest on a scale that displays weight in Newtons. Assume $g = 10.0\ m/s^2$, $\rho_{water} = 1.00 \times 10^3\ kg/m^3$. What is the weight reading on the scale?

1. i)   B   While in the boat, the anchor displaces an equal weight of water. Since the anchor is denser than the water (it sinks), the displaced volume of water is larger than the volume of the anchor. When the anchor is thrown overboard, it displaces only its own volume. Less water is displaced, so the water level drops.

   ii)   B   During an adiabatic process $Q = 0$. The first law of thermodynamics then implies that $\Delta E_{int} = -W$. In any expansion, the gas does positive work, so in this case the internal energy must decrease. Since $E_{int} = nC_V T$, the temperature must decrease. That is, the work done by the gas is done at the expense of internal energy of the gas since there is no heat exchange.

   iii)   C   The electric field between the capacitor plates is constant, independent of position. Thus the force and the acceleration on the proton are constant. Kinematics then gives us $v^2 = v_0^2 + 2ad$. Since the initial velocity is zero, the final velocity is proportional to the square root of the distance traveled.

   iv)   B   The lowest harmonic of a stretched string corresponds to $L = \lambda/2$. The lowest harmonic of a closed organ pipe corresponds to $L_{organ} = \lambda/4$. If the wavelengths are equal, then $L_{organ} = \lambda/4 = 2L/4 = L/2$.

   v)   B   The first quarter circle gives an electric field E_1 that points 45° below the x-axis, away from the middle of the arc. As you add pieces, you must add the electric field vectors for each piece. Two pieces (B) give $\sqrt{2}\ E_1$ pointing down, three pieces (C) give the same as one piece (A), since the two opposing pieces cancel, and four pieces (D) give zero field.

   vi)   A   The electric potential of a group of charges is proportional to the sum of q/r for each charge. In cases B and C the sum is always the same sign, so can only be zero at infinity. For case A, all points equidistant from the two charges will have $V = 0$.

   vii)   A   With all bulbs in place, the effective resistance of the two bulbs in parallel (1&2) is half the individual resistance R. Thus the voltage drop across the parallel pair is less than the drop across bulb 3, and so is less than half the battery voltage. When bulb 2 is removed, the voltage drop across bulb 1 becomes half the battery voltage (same as bulb 3 now). Since the voltage across bulb 1 increases, the power dissipated ($P = V^2/R$) increases.

2. a)   The work done in a cycle is the area enclosed by the cycle in the p-V diagram. In this case, we simply need to find the area of the rectangle:

$$W = \Delta p \Delta V = (3p_0 - p_0)(4V_0 - V_0)$$

$$W = 6p_0V_0 = 6\left(1.00 \times 10^5 \, Pa\right)\left(0.25m^3\right)$$

$$\boxed{W = 1.5 \times 10^5 \, J}$$

b)   Process ab is a constant volume process and process bc is a constant pressure process. The heat transferred is found using the appropriate specific heat. The temperature changes are found using $pV = nRT$.

$$Q_{abc} = Q_{ab} + Q_{bc}$$

$$Q_{abc} = nC_v\Delta T_{ab} + nC_p\Delta T_{bc} = nC_v\frac{\Delta p_{ab}V_0}{nR} + nC_p\frac{3p_0\Delta V_{bc}}{nR}$$

$$Q_{abc} = n\tfrac{3}{2}R\frac{(3p_0 - p_0)V_0}{nR} + n\tfrac{5}{2}R\frac{3p_0(4V_0 - V_0)}{nR}$$

$$Q_{abc} = \tfrac{3}{2}2p_0V_0 + \tfrac{5}{2}9p_0V_0 = 25.5p_0V_0 = 25.5(1.00 \times 10^5 \, Pa)(0.25m^3)$$

$$\boxed{Q_{abc} = 6.375 \times 10^5 \, J}$$

c)   The efficiency of the cycle is:

$$\varepsilon = \frac{W}{Q_{in}} = \frac{6p_0V_0}{25.5p_0V_0} = \frac{6}{25.5}$$

$$\boxed{\varepsilon = 23.5\%}$$

3. a) First find the resistance of the identical bulbs. Then find the equivalent resistances of the two groups of bulbs in series (1&2 and 3&4&5). The same battery voltage $V = 10V$ is dropped across each group. Let i_1 be the current through the first group and hence through bulb 1, and i_5 be the current through the second group and hence through bulb 5.

$$P = \frac{V^2}{R} \quad \Rightarrow \quad R = \frac{V^2}{P} = \frac{(10V)^2}{20W} = 5\Omega$$

$$R_{12} = R_1 + R_2 = 2R = 10\Omega$$

$$R_{345} = R_3 + R_4 + R_5 = 3R = 15\Omega$$

$$i_1 = \frac{V}{R_{12}} = \frac{10V}{10\Omega} = 1A$$

$$i_5 = \frac{V}{R_{345}} = \frac{10V}{15\Omega} = \tfrac{2}{3}A$$

$$\boxed{i_1 = 1.0A} \qquad\qquad \boxed{i_5 = 0.67A}$$

b) Before the switch is closed, there is a 5V drop across bulb 1 ($V_1 = i_1 R$). After the switch is closed, bulb 1 is in parallel with the series combination of bulbs 3 and 4. This parallel combination (1&(3&4)) is in series with the parallel combination 2&5. The 134 combination has a higher resistance and hence more voltage ($>5V$). Since the voltage on bulb 1 increases it burns brighter ($P = V^2/R$).

$$R_{34} = R_3 + R_4 = 2R = 10\Omega$$

$$\frac{1}{R_{134}} = \frac{1}{R_1} + \frac{1}{R_{34}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \quad \Rightarrow \quad R_{134} = \tfrac{2}{3}R$$

$$\frac{1}{R_{25}} = \frac{1}{R_2} + \frac{1}{R_5} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \Rightarrow \quad R_{25} = \tfrac{1}{2}R$$

$$R_{total} = R_{134} + R_{25} = \tfrac{7}{6}R \quad \Rightarrow \quad i_{total} = \frac{10V}{R_{total}} = \frac{10V}{\tfrac{7}{6}5\Omega} = 1.71A$$

$$V_1 = V_{134} = i_{total}R_{134} = \frac{10V}{\tfrac{7}{6}R}\tfrac{2}{3}R = \tfrac{4}{7}10V = 5.7V > 5V$$

$$\boxed{\text{Bulb 1 burns brighter}}$$

4. a) Find the capacitor that is equivalent to the two in series in the circuit. Note that the charge
on each capacitor is the same as on the equivalent capacitor.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2\mu F} + \frac{1}{4\mu F} = \frac{3}{4\mu F} \quad \Rightarrow \quad C_{eq} = \frac{4}{3}\mu F$$

$$q_1 = q_2 = q_{eq} = C_{eq}V_{eq} = \frac{4}{3}\mu F(12V) = 16\mu C$$

$$\boxed{q_1 = 16\mu C} \qquad\qquad \boxed{q_2 = 16\mu C}$$

b) The dielectric causes the capacitance C_2 to increase by the factor $\kappa = 3$. Follow the same
calculation to get:

$$\frac{1}{C'_{eq}} = \frac{1}{C_1} + \frac{1}{\kappa C_2} = \frac{1}{2\mu F} + \frac{1}{3(4\mu F)} = \frac{7}{12\mu F} \quad \Rightarrow \quad C'_{eq} = \frac{12}{7}\mu F$$

$$q'_1 = q'_2 = q'_{eq} = C'_{eq}V_{eq} = \frac{12}{7}\mu F(12V) = 20.6\mu C \quad \Rightarrow \quad V'_1 = \frac{q'_1}{C_1} = \frac{20.6\mu C}{2\mu F} = 10.3V$$

$$\boxed{q'_1 = 20.6\mu C} \qquad\qquad \boxed{q'_2 = 20.6\mu C}$$

c) From part (b), the voltage across the resistor immediately after closing the switch is
$V'_1 = 10.3$V. This voltage will drive a current through the resistor, which will increase the
charge on C_2. The voltage across C_2 will increase, but can reach a maximum of only 12V.
Thus after a long time the current must stop, at which point the voltage across R is 0V.

$$V'_1 = 10.3V$$

$$i(t=0) = \frac{V'_1}{R} = \frac{10.3V}{5\Omega}$$

$$\boxed{i(t=0) = 2.1A} \qquad\qquad \boxed{i(t=\infty) = 0A}$$

5. Consider the rod as composed of infinitesimal charge elements $dq = \lambda dx$, at position x from the origin. Each element produces an electric field dE at the origin that points in the negative x direction. To find the total field, integrate over the length of the rod. The charge density $\lambda = q/L$.

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{x^2}$$

$$E_x = \int dE_x = \int_d^{d+L} \frac{-1}{4\pi\varepsilon_0} \frac{\lambda dx}{x^2}$$

$$E_x = \frac{-\lambda}{4\pi\varepsilon_0} \int_d^{d+L} \frac{dx}{x^2}$$

$$E_x = \frac{-\lambda}{4\pi\varepsilon_0} \left[\frac{-1}{x} \right]_d^{d+L} = \frac{-\lambda}{4\pi\varepsilon_0} \left[\frac{-1}{d+L} - \frac{-1}{d} \right]$$

$$E_x = \frac{-\lambda}{4\pi\varepsilon_0} \left[\frac{-d}{d(d+L)} + \frac{d+L}{d(d+L)} \right]$$

$$E_x = \frac{-\lambda}{4\pi\varepsilon_0} \frac{L}{d(d+L)}$$

$$\boxed{E_x = \frac{-1}{4\pi\varepsilon_0} \frac{q}{d(d+L)}}$$

6. Consider the wire as three pieces, the left and right straight sections and the semicircle. Calculate the potential due to each and then sum to find the total potential. Note that the left and right sections will have equal contributions to the potential at the origin.

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$V_{right} = \int_R^{R+L}\frac{\lambda dx}{4\pi\varepsilon_0 x} = \frac{\lambda}{4\pi\varepsilon_0}\int_R^{R+L}\frac{dx}{x}$$

$$V_{right} = \frac{\lambda}{4\pi\varepsilon_0}[\ln x]_R^{R+L} = \frac{\lambda}{4\pi\varepsilon_0}[\ln(R+L)-\ln R] = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{R+L}{R}\right)$$

$$V_{left} = V_{right}$$

$$V_{semi} = \int_0^\pi \frac{\lambda R d\theta}{4\pi\varepsilon_0 R}$$

$$V_{semi} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^\pi d\theta = \frac{\lambda}{4\pi\varepsilon_0}[\theta]_0^\pi = \frac{\lambda\pi}{4\pi\varepsilon_0}$$

$$V_{total} = V_{left} + V_{right} + V_{semi} = 2V_{right} + V_{semi}$$

$$V_{total} = 2\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{R+L}{R}\right) + \frac{\lambda\pi}{4\pi\varepsilon_0}$$

$$V_{total} = \frac{\lambda}{4\pi\varepsilon_0}\left[2\ln\left(\frac{R+L}{R}\right)+\pi\right] = \frac{4.0\,\mu C/m}{4\pi(8.85\times10^{-12}\,C^2/Nm^2)}\left[2\ln\frac{8}{5}+\pi\right]$$

$$\boxed{V_{total} = 1.47\times10^5\,V}$$

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7. Archimedes' principle states that the buoyant force on the cube is equal to the weight of the water displaced by the cube.

$$F_{buoyant} = \rho_{water}gd^3$$

$$F_{buoyant} = 1.00\times10^3\,kg/m^3\,(10\,m/s^2)(0.300m)^3$$

$$\boxed{F_{buoyant} = 270N}$$

Since this force acts upward on the block, an equal and opposite force acts on the water, increasing the measured weight on the scale:

$$W_{scale} = m_{water+bucket}g + F_{buoyant}$$

$$W_{scale} = 8.00kg(10.0\,m/s^2) + 270N$$

$$\boxed{W_{scale} = 350N}$$