- Exam is closed book, closed notes. Use only the provided formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- **Show all your work and explain your reasoning** (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.
- Charges labeled +*q* are positive and those labeled -*q* are negative.

1. (35 points) For each of these multiple choice questions, indicate the correct response (A, B, or C) on the page for problem 1 in your exam booklet.

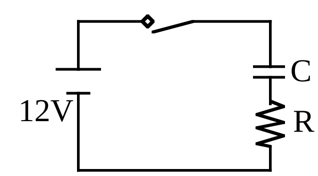
i) A boat made entirely of wood floats in a large swimming pool. Does the water level in the pool move upward, move downward, or remain the same when the boat is disassembled and all the wooden pieces are floating individually in the pool?

A) Upward. B) Downward. C) Remains the same.

ii) An ideal gas undergoes an isobaric (constant pressure) expansion. Does the temperature of the gas increase, decrease, or remain the same?

A) Increases. B) Decreases. C) Remains the same.

iii) A capacitor *C* is charged up using the circuit shown at right. If the resistance of the resistor *R* is increased, does the time for the capacitor to acquire a certain charge after the switch is closed increase, decrease, or remain the same?

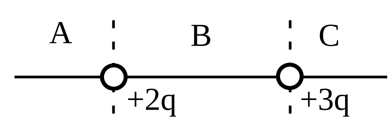


A) Increases. B) Decreases. C) Remains the same.

iv) The equations below describe three waves. Which wave has the largest speed?

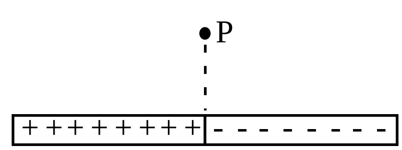
A) $y_A = 2\cos(3x - 4t)$ B) $y_B = 3\cos(4x + 3t)$ C) $y_C = 4\cos(2x - 3t)$

v) Two charged particles are fixed in place on a line as shown at right. In which region on the line can a proton be placed such that the proton will remain at rest?



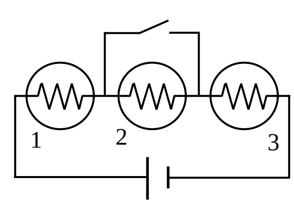
A) Region A B) Region B C) Region C

vi) A thin insulating rod has a uniformly distributed charge +*q* on its left half and -*q* on its right half. Consider a point P on the perpendicular bisector of the rod, as shown at right. If the electric potential is defined to be zero at infinity, then is the electric potential at point P positive, negative, or zero?



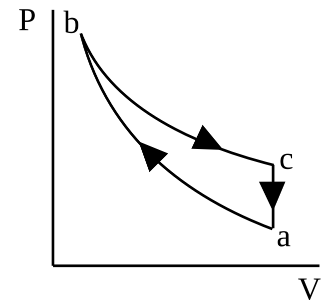
A) Positive B) Negative C) Zero

vii) Three identical light bulbs are connected in the circuit shown at right. When the switch is closed, does the brightness of bulb 1 increase, decrease, or remain the same?

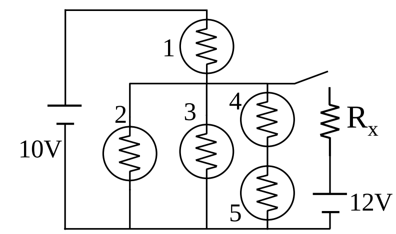


A) Increases. B) Decreases. C) Remains the same.

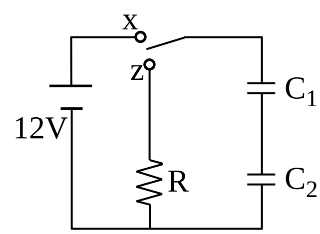
2. (35 points) Five moles of an ideal monatomic gas are taken through the cycle *abca* shown at right. Process *ab* is adiabatic and process *bc* is isothermal. Assume $P_b = 2.0 \times 10^5$ Pa, $V_b = 0.50\ m^3$, $V_c = 6\ V_b$ and $R = 8.3\ J/molK$.
   a) How much work is done by the gas during the complete cycle?
   b) What is the efficiency of this heat engine?
   c) Is the efficiency found in part (b) larger, smaller, or the same as the efficiency of an ideal engine operating between the highest and lowest temperatures that occur in the cycle?
   d) Find the entropy change of the gas during the process *ca*.



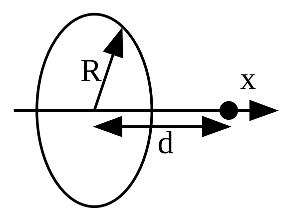
3. (35 points) Five identical light bulbs are connected in a circuit as shown at right. Each light bulb dissipates 25 *W* of electrical power when a 10 *V* electrical potential difference is placed across the bulb. The switch is open for parts (a) and (b).
   a) Find the current through light bulbs 1, 3, and 5.
   b) Light bulb 5 is replaced by a light bulb that is normally brighter (*i.e.*, when placed across 10 *V*). Does bulb 2 get brighter, dimmer, or remain the same, and why?
   c) When the switch is closed, light bulb 1 turns off. What is the value of the resistor R_x?



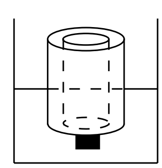
4. (30 points) Two capacitors (C_1 = 3 μF, C_2 = 6 μF) are connected in a circuit as shown at right. (Switch is set to x for (a) and (b)).
   a) Find the magnitude of the voltage across each capacitor.
   a) Find the magnitude of the voltage across each capacitor when a dielectric material of dielectric constant κ = 4 is inserted in capacitor 1.
   c) With the dielectric in place, move the switch to z to connect the resistor *R* = 5 Ω. Find the current through the resistor immediately after moving the switch and then a long time later.



5. (35 points) A thin hoop of radius *R* and uniform linear charge density λ has its center at the origin of the *x*-axis and its axis coincident with the *x*-axis, as shown at right. Answer all parts in terms of the parameters given and any other physical constants required.
   a) Find the electric potential on the *x*-axis at a distance *d* from the origin (assuming the potential is zero at infinity).
   b) Find the electric field on the *x*-axis at a distance *d* from the origin.
   c) Find the location of an additional point charge -*Q* (oppositely charged compared to the hoop) that makes the electric field at the position in (b) become zero.



6. (30 points) A hollow styrofoam (density $\rho_S = 0.2\ g/cm^3$) cylinder has length $L = 20\ cm$, inner radius $r_1 = 3.0\ cm$, and outer radius $r_2 = 5.0\ cm$. A small weight of mass $m_0 = 60\ g$ and volume $V_0 = 10\ cm^3$ is attached to the bottom of the cylinder and the system floats in water (density $\rho_W = 1.0\ g/cm^3$). Assume that the cylinder floats with the axis vertical as shown.
   a) Find the distance from the water level to the top of the cylinder.
   b) Oil (density $\rho_{oil} = 0.5\ g/cm^3$) is then added to the system (same level inside and outside cylinder) until a tuning fork of frequency $f = 2125\ Hz$ is resonant with the third harmonic of the air column in the cylinder. Find the depth of the oil layer. Assume the speed of sound in air is 340 *m/s*.



1. i) C When the boat is floating, the displaced water has the same weight as the total weight of the boat. When the individual pieces float, the same weight must be supported, so the same amount of water must be displaced.

ii) A During an isobaric process the pressure is constant. The ideal gas law ($pV = nRT$) then implies that T is proportional to V. Thus if the gas expands, it must get hotter.

iii) A Increasing the resistance decreases the current, which means it takes longer for the charge to accumulate on the capacitor.

iv) C The waves are written in the form $y = y_{max} \cos(kx \pm \omega t)$. The speed of a wave is given by $v = \omega/k$. This ratio is largest for wave C. The sign tells us which way the wave travels.

v) B A positively charged proton is repelled by the $+2q$ charge and by the $+3q$ charge. In regions A and C these repulsions will add to force the proton to infinity. In region B the repulsions can cancel at the appropriate place to give no net force.

vi) C Since P is equally spaced from the two sides and the potential is proportional to dq/r for each charge element, the two sides will cancel due to their opposite charges. Note that the electric field is not zero, just the potential.

vii) A The switch short circuits bulb 2, reducing the total resistance of the circuit from $3R$ to $2R$. The current ($i = V/R_{total}$) thus increases, which causes the power dissipated in each resistor ($P = i^2R$) to increase.

2. a) The work done in a cycle is the area enclosed by the cycle in the *p*-*V* diagram. There is no contribution from *ca*, we integrate along the isotherm *bc*, and we use the 1st law of thermodynamics along the adiabat *ab*. To find the temperatures, relate the points using ideal gas law and adiabat definition.

$$W = W_{ab} + W_{bc}$$

$$W_{ab} = -\Delta E_{ab} = -nC_V \Delta T_{ab} = -\tfrac{3}{2} nR\left(T_b - T_a\right)$$

$$W_{bc} = \int_b^c p\,dV = nRT_b \int_{V_b}^{V_c} \frac{dV}{V} = nRT_b \ln\frac{V_c}{V_b}$$

$$W = nRT_b \ln\frac{V_c}{V_b} - \tfrac{3}{2} nR\left(T_b - T_a\right)$$

$$T_b: \quad P_b V_b = nRT_b \quad \Rightarrow \quad T_b = \frac{P_b V_b}{nR} = \frac{2\times 10^5\,Pa\left(0.5m^3\right)}{5mol\left(8.3\,J/molK\right)} = 2410K = T_c \text{ (isotherm)}$$

$$ab: \quad P_a V_a^\gamma = P_b V_b^\gamma \quad \Rightarrow \quad P_a = P_b\left(\frac{V_b}{V_a}\right)^\gamma = P_b\left(\tfrac{1}{6}\right)^\gamma \quad \Rightarrow \quad T_a = \frac{P_a V_a}{nR} = \frac{P_b 6V_b}{nR}\left(\tfrac{1}{6}\right)^{\frac{5}{3}} = 6\left(\tfrac{1}{6}\right)^{\frac{5}{3}} T_b = 730K$$

$$W = 5mol\left(8.3\,J/molK\right)\left(2410K \ln 6 - \tfrac{3}{2}\left(2410K - 730K\right)\right)$$

$$\boxed{W = 74.6kJ}$$

b) Heat is transferred into the gas during the process *bc*, which is isothermal ($\Delta E_{bc} = 0$).

$$Q_{bc} = W_{bc} = nRT_b \ln\frac{V_c}{V_b}$$

$$Q_{bc} = nRT_b \ln 6 = 5mol\left(8.3\,J/molK\right)2410K(1.79) = 179kJ$$

$$\varepsilon = \frac{W}{Q_{in}} = \frac{74.6kJ}{179kJ}$$

$$\boxed{\varepsilon = 42\%}$$

c) The highest and lowest temperatures in the cycle occur at *b* and *a*, respectively. Thus the efficiency of an ideal engine would be:

$$\varepsilon_{ideal} = 1 - \frac{T_C}{T_H} = 1 - \frac{T_a}{T_b} = 1 - \frac{730K}{2410K}$$

$$\boxed{\varepsilon_{ideal} = 70\% \qquad \Rightarrow \qquad \varepsilon < \varepsilon_{ideal}}$$

d) Process *ca* is a constant volume process, so the heat transferred is $Q = nC_V\Delta T$. The entropy change is thus:

$$\Delta S = \int_c^a \frac{dQ}{T} = \int_{T_c}^{T_a} \frac{nC_V dT}{T} = nC_V \int_{T_c}^{T_a} \frac{dT}{T} = nC_V \ln\frac{T_a}{T_c}$$

$$\Delta S = n\tfrac{3}{2}R\ln\frac{730K}{2410K} = 5mol\,\frac{3}{2}(8.3\,J/molK)(-1.19)$$

$$\boxed{\Delta S = -74.3\,J/K}$$

3. a) First find the resistance of the identical bulbs. Then find the equivalent resistances of the various groups, calling the lower 4 group "low."

$$P = \frac{V^2}{R} \quad \Rightarrow \quad R = \frac{V^2}{P} = \frac{(10V)^2}{25W} = 4\Omega$$

$$R_{45} = R_4 + R_5 = 2R = 8\Omega$$

$$\frac{1}{R_{low}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R} = \frac{5}{2R}$$

$$R_{low} = 0.4R = 1.6\Omega$$

$$R_{total} = R_{up} + R_{low} = R + 0.4R = 1.4R = 5.6\Omega$$

$$i_{total} = \frac{V}{R_{total}} = \frac{10V}{5.6\Omega} = 1.79A$$

$$i_1 = i_{total} = 1.79A \quad \Rightarrow \quad V_1 = i_1 R = 1.79A(4\Omega) = 7.16V$$

$$V_{low} = 10V - V_1 = 10V - 7.16V = 2.84V = V_3 = V_{45}$$

$$i_3 = \frac{V_3}{R_3} = \frac{2.84V}{4\Omega} = 0.71A$$

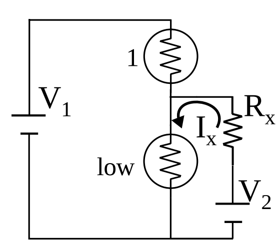
$$i_5 = i_4 = \frac{V_{45}}{R_{45}} = \frac{2.84V}{8\Omega} = 0.36A$$

$\boxed{i_1 = 1.79A}$ $\boxed{i_3 = 0.71A}$ $\boxed{i_5 = 0.36A}$

b) If the new bulb is normally brighter, then its resistance must be smaller, since $P = V^2/R$. So replace R_5 by a smaller resistance in the above calculations. A smaller R_5 will give a smaller R_{low} and hence a smaller R_{total}. This will mean a larger total current, which implies a larger voltage drop across R_1 and a correspondingly smaller voltage drop across the lower set. Hence, lightbulb 2 has a smaller voltage drop and will thus burn less brightly since $P = V^2/R$.

[Bulb 2 gets dimmer]

c) If light bulb 1 turns off, then its current and voltage drop must be zero. All the current (I_x) flows through R_x and R_{low}. Apply Kirchoff's loop rule as shown.



$$V_2 - I_x R_x - I_x R_{low} = 0$$

$$V_1 - I_x R_{low} = 0$$

$$I_x R_x = V_2 - I_x R_{low} = V_2 - V_1$$

$$R_x = \frac{V_2 - V_1}{I_x} = \frac{V_2 - V_1}{V_1 / R_{low}} = R_{low}\frac{V_2 - V_1}{V_1} = 1.6\Omega\,\frac{12V - 10V}{10V}$$

$\boxed{R_x = 0.32\Omega}$

4. a) Find the equivalent capacitance of the two capacitors in series. Then find the charge on the equivalent capacitor and note that the same charge resides on each of the capacitors in series.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3\mu F} + \frac{1}{6\mu F} = \frac{3}{6\mu F} \quad \Rightarrow \quad C_{eq} = 2\mu F$$

$$q_{eq} = q_1 = q_2 = C_{eq}V_{eq} = 2\mu F(12V) = 24\mu C$$

$$V_1 = \frac{q_1}{C_1} = \frac{24\mu C}{3\mu F} = 8V$$

$$V_2 = \frac{q_2}{C_2} = \frac{24\mu C}{6\mu F} = 4V$$

$$\boxed{V_1 = 8V} \qquad \boxed{V_2 = 4V}$$

b) The dielectric causes the capacitance C_1 to increase by the factor $\kappa = 4$. Follow the same calculation to get:

$$\frac{1}{C_{eq}} = \frac{1}{\kappa C_1} + \frac{1}{C_2} = \frac{1}{4(3\mu F)} + \frac{1}{6\mu F} = \frac{3}{12\mu F} \quad \Rightarrow \quad C_{eq} = 4\mu F$$

$$q_{eq} = q_1 = q_2 = C_{eq}V_{eq} = 4\mu F(12V) = 48\mu C$$

$$V_1 = \frac{q_1}{\kappa C_1} = \frac{48\mu C}{12\mu F} = 4V$$

$$V_2 = \frac{q_2}{C_2} = \frac{48\mu C}{6\mu F} = 8V$$

$$\boxed{V_1 = 4V} \qquad \boxed{V_2 = 8V}$$

c) Since the capacitors are fully charged, the voltage across the resistor immediately after moving the switch is $V = 12\text{V}$. This voltage will drive a current through the resistor, which will deplete the charges on the capacitors. Thus after a long time the current must stop, at which point the voltage across R is 0V.

$$i(t=0) = \frac{V}{R} = \frac{12V}{5\Omega}$$

$$\boxed{i(t=0) = 2.4A} \qquad \boxed{i(t=\infty) = 0A}$$

5. a) Consider the hoop as composed of infinitesimal charge elements dq. Each element produces an electric potential dV at the point P on the x-axis. To find the total potential, integrate over the length of the hoop. Since each element of the hoop is the same distance from the axis, the denominator can be pulled out of the integral.

$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{R^2 + d^2}} = \frac{k_e}{\sqrt{R^2 + d^2}} \int dq$$

$$V = \frac{k_e Q_{hoop}}{\sqrt{R^2 + d^2}} = \frac{k_e 2\pi R\lambda}{\sqrt{R^2 + d^2}}$$

$$\boxed{V = \frac{k_e 2\pi R\lambda}{\sqrt{R^2 + d^2}}}$$

b) The field on the axis has only an x-component, by symmetry, and can be found by differentiating the potential, if we replace d by the variable x for the derivative:

$$V(x) = \frac{k_e 2\pi R\lambda}{\sqrt{R^2 + x^2}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{k_e 2\pi R\lambda}{\left(R^2 + x^2\right)^{\frac{3}{2}}}\left(-\frac{1}{2}\right)(2x) = \frac{k_e 2\pi R\lambda x}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

$$\boxed{E_x = \frac{k_e 2\pi R\lambda d}{\left(R^2 + d^2\right)^{\frac{3}{2}}}}$$

c) A new charge -Q must produce a field that exactly cancels the field above. Since -Q is opposite in sign to Q_{hoop}, the force will be attractive. Hence Q must be placed on the x-axis at a position $x < d$.

$$E_x = \frac{k_e 2\pi R\lambda d}{\left(R^2 + d^2\right)^{\frac{3}{2}}} - \frac{k_e Q}{(d - x)^2} = 0$$

$$(d - x)^2 = \frac{Q\left(R^2 + d^2\right)^{\frac{3}{2}}}{2\pi R\lambda d}$$

$$\boxed{x = d - \sqrt{\frac{Q\left(R^2 + d^2\right)^{\frac{3}{2}}}{2\pi R\lambda d}}}$$

6. a) Let the distance from the water level to the top of the cylinder be *x*. Archimedes' principle states that the buoyant force is equal to the weight of the displaced fluid. Since the cylinder is floating, the buoyant force is equal to the weight of the cylinder and the attached weight.

$$F_{buoyant} = m_{total} g$$

$$\rho_w g V_0 + \rho_w g \pi (L - x)\left(r_2^2 - r_1^2\right) = m_0 g + \rho_S g \pi L\left(r_2^2 - r_1^2\right)$$

$$\rho_w \pi (L - x)\left(r_2^2 - r_1^2\right) = m_0 + \rho_S \pi L\left(r_2^2 - r_1^2\right) - \rho_w V_0$$

$$(L - x) = \frac{m_0 + \rho_S \pi L\left(r_2^2 - r_1^2\right) - \rho_w V_0}{\rho_w \pi\left(r_2^2 - r_1^2\right)}$$

$$x = L - \frac{m_0 + \rho_S \pi L\left(r_2^2 - r_1^2\right) - \rho_w V_0}{\rho_w \pi\left(r_2^2 - r_1^2\right)}$$

$$r_2^2 - r_1^2 = (5cm)^2 - (3cm)^2 = 25cm^2 - 9cm^2 = 16cm^2$$

$$x = 20cm - \frac{60g + \pi\left(0.2\, g/cm^3\right)20cm\left(16cm^2\right) - 1.0\, g/cm^3\left(10cm^3\right)}{\pi\left(1.0\, g/cm^3\right)\left(16cm^2\right)}$$

$$x = 20cm - 5.0cm$$

$$\boxed{x = 15cm}$$

b) An air column with one end open and one end closed has a quarter-wave fundamental and odd integer harmonics. Let the air column height from the oil level to the top of the cylinder be *d*, and the thickness of the oil level be *z*.

$$d = \frac{3}{4}\lambda = \frac{3}{4}\frac{v}{f} = \frac{3}{4}\frac{340\, m/s}{2125Hz} = 0.12m = 12cm$$

$$F_{buoyant} = m_{total} g$$

$$\rho_w g V_0 + \rho_w g \pi (L - d - z)\left(r_2^2 - r_1^2\right) + \rho_{oil} g \pi z\left(r_2^2 - r_1^2\right) = m_0 g + \rho_S g \pi L\left(r_2^2 - r_1^2\right)$$

$$\left(\rho_w - \rho_{oil}\right)\pi z\left(r_2^2 - r_1^2\right) = \rho_w V_0 + \left(\rho_w - \rho_S\right)\pi L\left(r_2^2 - r_1^2\right) - \rho_w \pi d\left(r_2^2 - r_1^2\right) - m_0$$

$$z = \frac{\rho_w V_0 - m_0 + \pi\left(r_2^2 - r_1^2\right)\left[\left(\rho_w - \rho_S\right)L - \rho_w d\right]}{\left(\rho_w - \rho_{oil}\right)\pi\left(r_2^2 - r_1^2\right)}$$

$$z = \frac{10g - 60g + \pi\left(16cm^2\right)\left[\left(0.8\, g/cm^3\right)20cm - \left(1.0\, g/cm^3\right)12cm\right]}{\pi\left(0.5\, g/cm^3\right)\left(16cm^2\right)}$$

$$\boxed{z = 6.0cm}$$